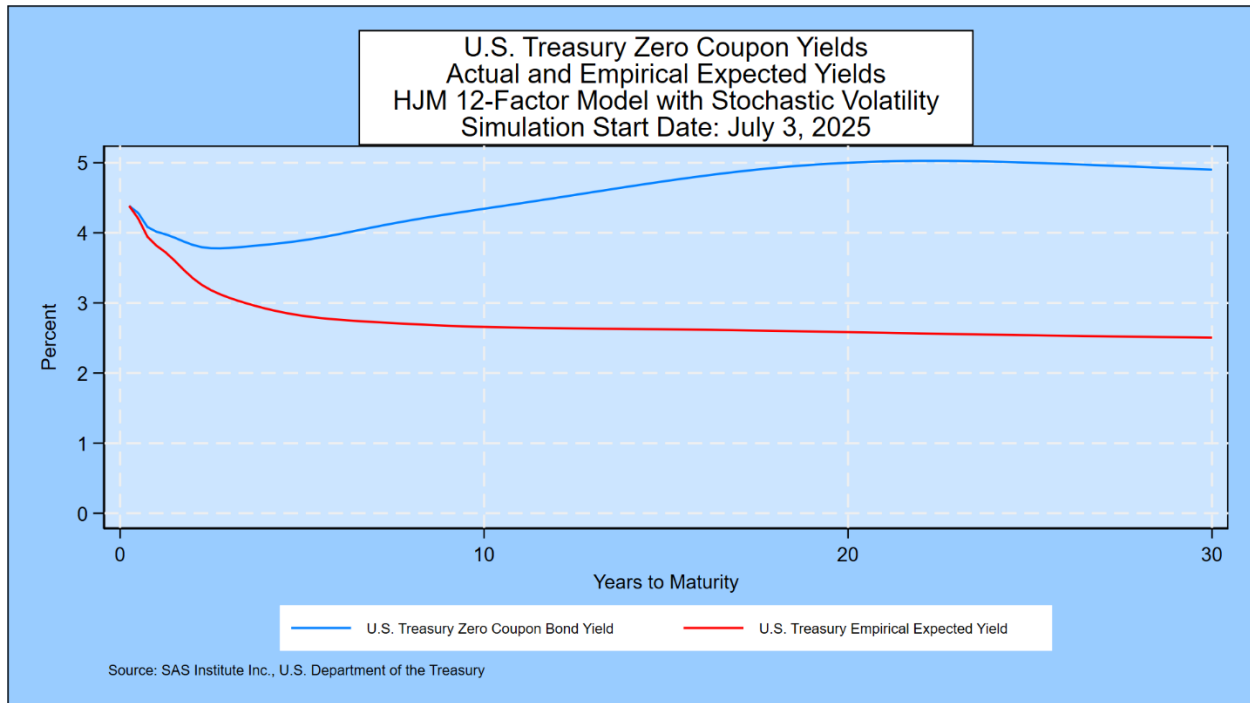


## **SAS Weekly Treasury Simulation, July 3, 2025: Most Likely Level for 3-Month Bill Rate in Ten Years Steady at the 1% to 2% Range**

### **Summary**

- The most likely range for 3-month bill yields is the 1% to 2% range, unchanged from last week. The probability of being in this range is 0.13% higher than the probability of the 0% to 1% range.
- Treasury 2-year yields moved to 3.88% this week from 3.73% last week. At 10 years, this week's yield is 4.35%, compared with 4.51% last week.
- As a result, the current 2-year/10-year Treasury spread is now 0.47% compared to 0.56% last week.
- The maximum probability that the 2-year/10-year Treasury spread is negative in the coming ten years is 25.2% in the 91-day period ending June 16, 2039, compared to 25.3% last week.
- The long-term peak in 1-month forward Treasuries is now 5.93% and well above the shortest maturity forward rate at 4.35%. Last week's peak was 5.98%. The longest maturity 1-month forward rate is now 4.36% versus 4.39% last week.

As explained in Prof. Robert Jarrow's book cited below, forward rates contain a risk premium above and beyond the market's expectations for the 3-month forward rate. We document the size of that risk premium in this graph, which shows the zero-coupon yield curve implied by current Treasury prices compared with the annualized compounded yield on 3-month Treasury bills that market participants would expect based on the daily movement of government bond yields in 14 countries since 1962. The risk premium, the reward for a long-term investment, is large and widens over most of the full 30-year maturity range. The graph also shows a decline in expected yields at a steady pace for the full 30 years. We explain the details below.



For more on this topic, see the analysis of government bond yields in 14 countries through May 31, 2025 given in the appendix.

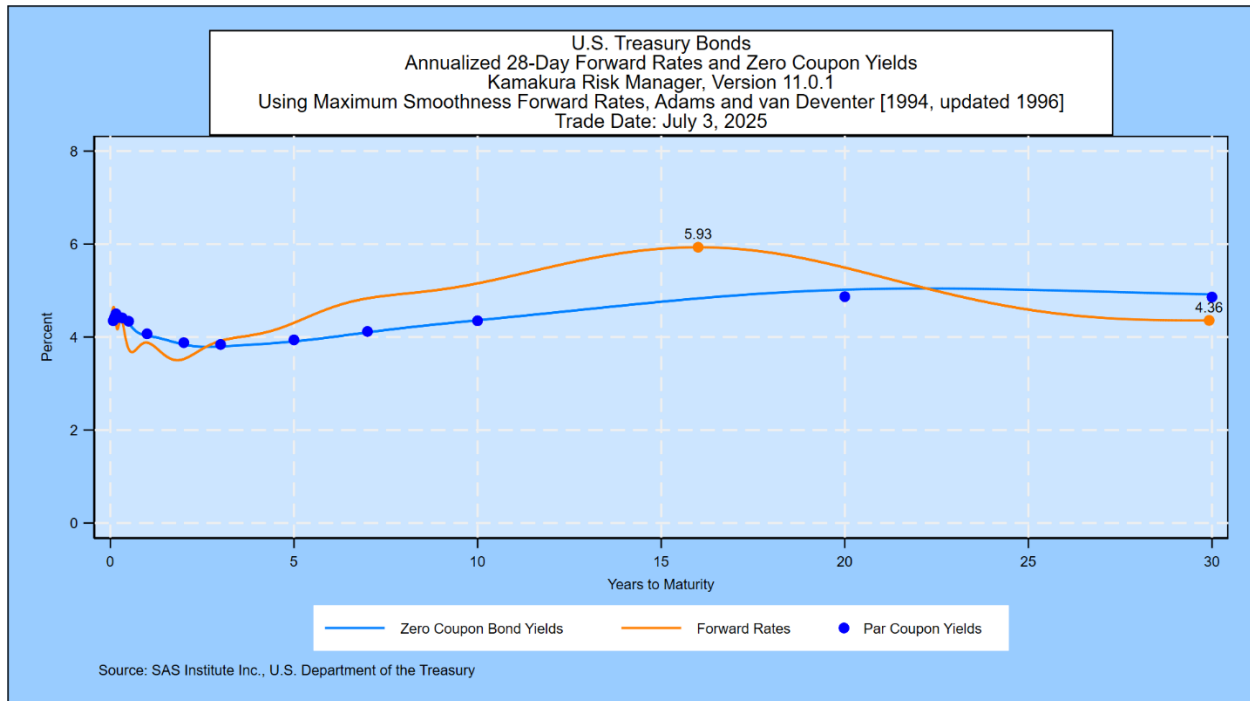
### **Inverted Yields, Negative Rates, and U.S. Treasury Probabilities 10 Years Forward**

The negative 2-year/10-year Treasury spread ended on August 26, 2024 after 537 trading days. The spread is currently at positive 0.47%. The table below shows that the August 26, 2024 streak of inverted yield curves is the longest in the U.S. Treasury market since the 2-year Treasury yield was first reported on June 1, 1976. The second longest streak is 423 trading days starting on August 18, 1978.

Date Rank	Start Date	End Date	Number of Trading Days	Date Rank	Start Date	End Date	Number of Trading Days
1	August 18, 1978	May 1, 1980	423	24	July 21, 1998	July 27, 1998	5
2	September 12, 1980	October 23, 1981	278	25	February 2, 2000	February 9, 2000	6
3	October 27, 1981	October 27, 1981	1	26	February 11, 2000	December 26, 2000	220
4	October 29, 1981	October 29, 1981	1	27	December 28, 2000	December 28, 2000	1
5	November 5, 1981	November 5, 1981	1	28	December 27, 2005	December 27, 2005	1
6	December 28, 1981	December 28, 1981	1	29	December 29, 2005	December 30, 2005	2
7	January 14, 1982	January 14, 1982	1	30	January 31, 2006	March 7, 2006	25
8	January 18, 1982	January 18, 1982	1	31	March 21, 2006	March 29, 2006	7
9	January 20, 1982	May 19, 1982	83	32	June 8, 2006	June 28, 2006	15
10	May 21, 1982	May 25, 1982	3	33	June 30, 2006	July 26, 2006	18
11	June 2, 1982	June 2, 1982	1	34	August 2, 2006	August 3, 2006	2
12	June 4, 1982	July 9, 1982	25	35	August 7, 2006	August 7, 2006	1
13	July 13, 1982	July 16, 1982	4	36	August 14, 2006	August 15, 2006	2
14	December 13, 1988	December 21, 1988	7	37	August 17, 2006	March 20, 2007	147
15	December 27, 1988	December 28, 1988	2	38	May 3, 2007	May 21, 2007	13
16	January 4, 1989	June 29, 1989	123	39	May 30, 2007	June 5, 2007	5
17	August 11, 1989	September 13, 1989	23	40	August 27, 2019	August 29, 2019	3
18	September 18, 1989	October 11, 1989	17	41	April 1, 2022	April 4, 2022	2
19	November 3, 1989	November 6, 1989	2	42	July 6, 2022	August 26, 2024	537
20	March 8, 1990	March 29, 1990	16	43	September 3, 2024	September 3, 2024	1
21	May 26, 1998	May 26, 1998	1	44	September 5, 2024	September 5, 2024	1
22	June 9, 1998	June 11, 1998	3				
23	June 15, 1998	July 9, 1998	18				

In this week's forecast, the focus is on three elements of interest rate behavior: the future probability of the recession-predicting inverted yield curve, the probability of negative rates, and the probability distribution of U.S. Treasury yields over the next decade.

We start from the closing U.S. Treasury yield curve published daily by the U.S. Department of the Treasury. Using a maximum smoothness forward rate approach, Friday's implied forward rate curve shows the shortest maturity 1-month forward rates at 4.35%, versus 4.19% last week. After the initial peak, there is a decline followed by a gradual rise until rates peak again at 5.93%, compared to 5.98% the previous week. Forward rates stabilize at 4.36% at the 30-year horizon, versus 4.39% last week.

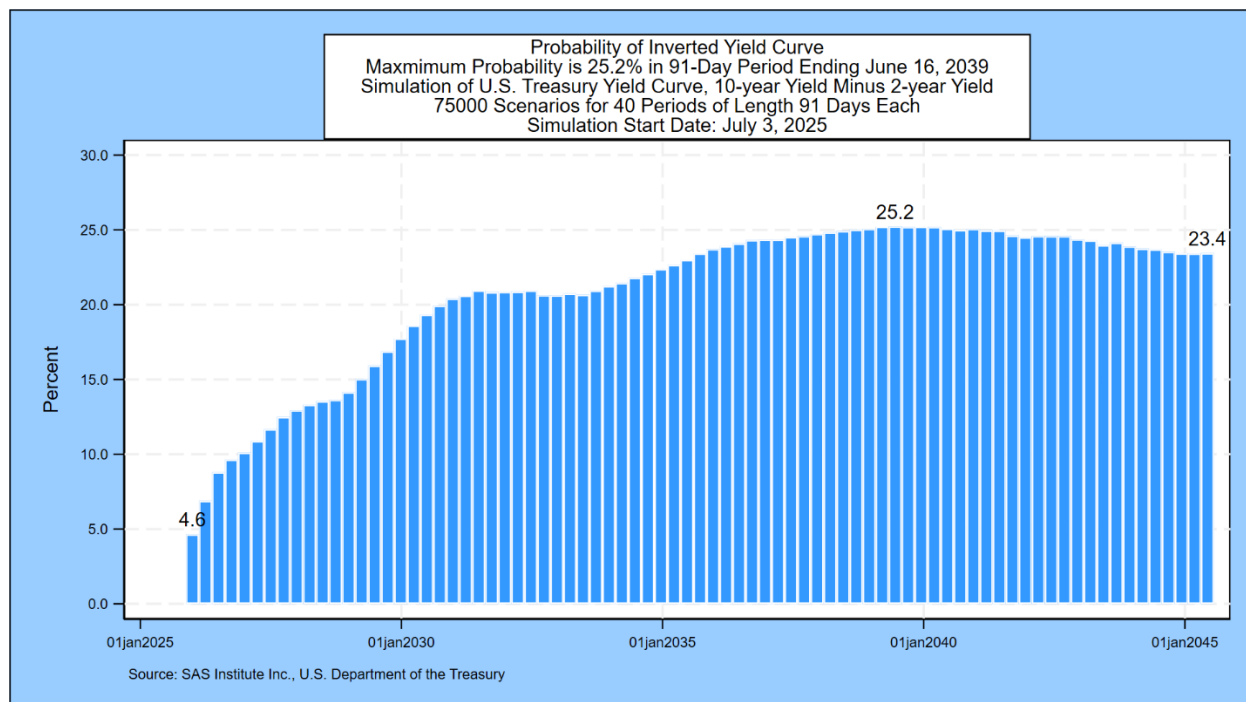


Using the methodology outlined in the appendix, we simulate 75,000 future paths for the U.S. Treasury yield curve out to thirty years. The next three sections summarize our conclusions from that simulation.

### **Inverted Treasury Yields: Positive Spread Now, 25.2% Probability by June 16, 2039**

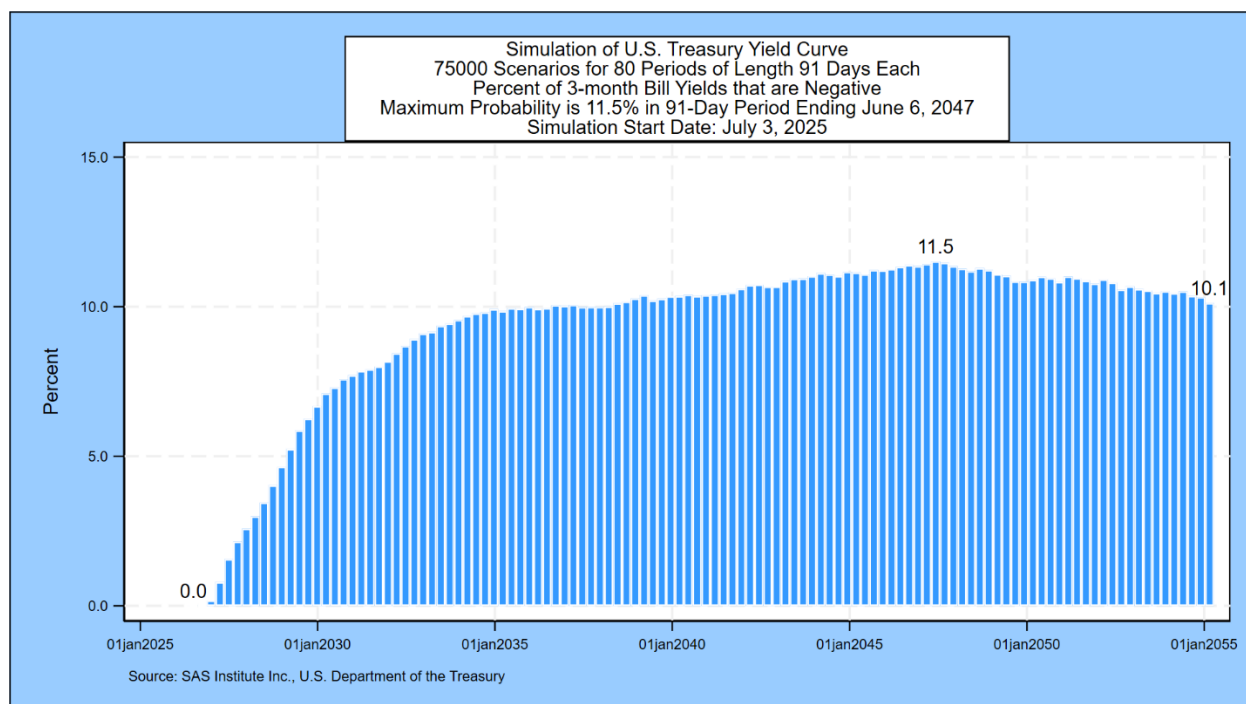
Many economists have concluded that a downward sloping U.S. Treasury yield curve is an important indicator of future recessions. A recent example is this paper by [Alex Domash and Lawrence H. Summers](#). We measure the probability that the 10-year par coupon Treasury yield is lower than the 2-year par coupon Treasury for every scenario in each of the first 40 quarterly periods in the simulation.<sup>1</sup> The next graph shows that the probability of an inverted yield now peaks at 25.2%, compared to 25.3% one week before, in the 91-day period ending June 16, 2039.

<sup>1</sup> After the first 20 years in the simulation, the 10-year Treasury cannot be derived from the initial 30 years of Treasury yields.



### Negative Treasury Bill Yields: 11.5% Probability by June 6, 2047

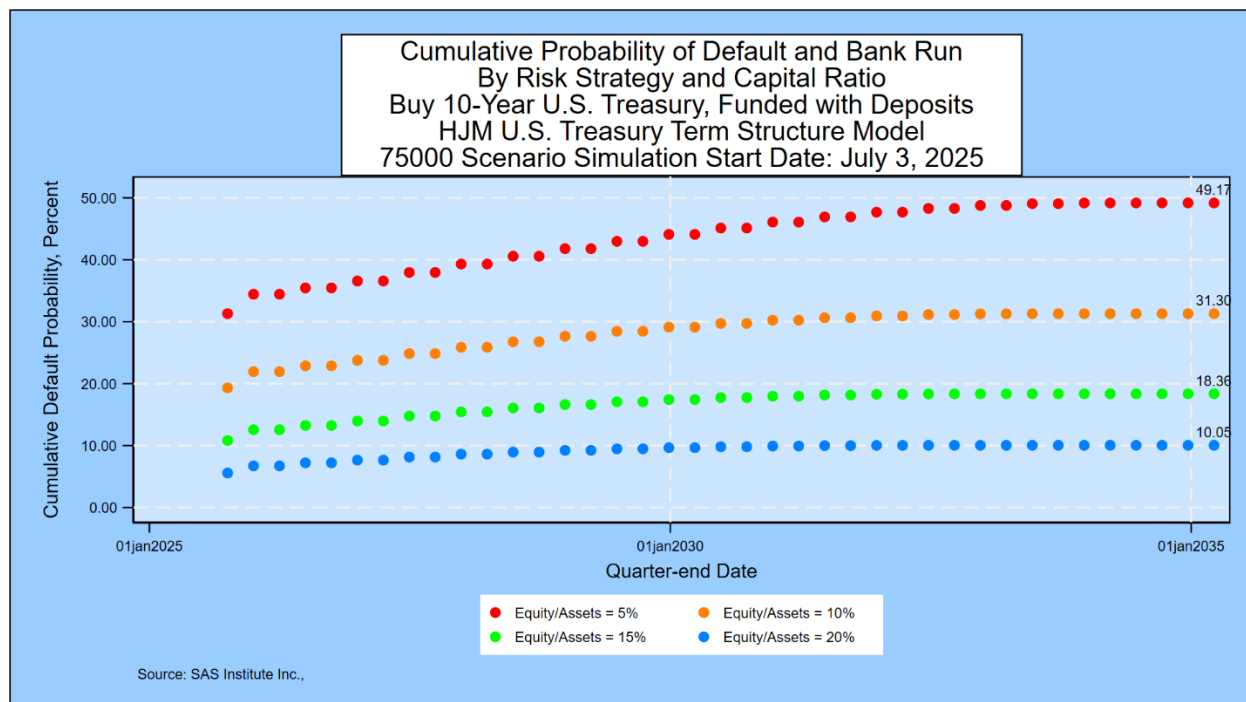
The next graph describes the probability of negative 3-month Treasury bill rates for all but the first 3 months of the next three decades. The probability of negative rates starts near zero and then peaks at 11.5%, compared to 11.5% last week, in the 91-day period ending June 6, 2047.



## Calculating the Default Risk from Interest Rate Maturity Mismatches

In light of the interest-rate-risk-driven failure of Silicon Valley Bank on March 10, 2023, we have added a table that applies equally well to banks, institutional investors, and individual investor mismatches from buying long-term Treasury bonds with borrowed short-term funds. We assume that the sole asset is a 10-year Treasury bond purchased at time zero at par value of \$100. We analyze default risk for four different initial market value of equity to market value of asset ratios: 5%, 10%, 15%, and 20%. For the banking example, we assume that the only class of liabilities is deposits that can be withdrawn at par at any time. In the institutional and retail investor case, we assume that the liability is essentially a borrowing on margin/repurchase agreement with the possibility of margin calls. For all investors, the amount of liabilities (95, 90, 85 or 80) represents a “strike price” on a put option held by the liability holders. Failure occurs via a margin call, bank run, or regulatory takeover (in the banking case) when the value of assets falls below the value of liabilities.

The graph below shows the cumulative 10-year probabilities of failure for each of the 4 possible capital ratios when the asset’s maturity is 10 years. For the 5 percent case, that default probability is 49.17%, compared to 49.78% from last week.



This default probability analysis is updated weekly based on the U.S. Treasury yield simulation described in the next section. The calculation process is the same for any portfolio of assets with credit risk included.

### U.S. Treasury Probabilities 10 Years Forward

In this section, the focus turns to the decade ahead. This week's simulation shows that the most likely range for the 3-month U.S. Treasury bill yield in ten years is from 1% to 2%, up one percent from last week. There is a 21.85% probability that the 3-month yield falls in this range, a change from 21.75% one week before. For the 10-year Treasury yield, the most likely range is from 2% to 3%, unchanged from last week. The probability of being in this range is 21.32%, compared to 21.28% one week prior.

In a [recent post on SeekingAlpha](#), we pointed out that a forecast of “heads” or “tails” in a coin flip leaves out critical information. What a sophisticated bettor needs to know is that, on average for a fair coin, the probability of heads is 50%. A forecast that the next coin flip will be “heads” is literally worth nothing to investors because the outcome is purely random.

The same is true for interest rates.

In this section we present the detailed probability distribution for both the 3-month Treasury bill rate and the 10-year U.S. Treasury yield 10 years forward using semi-annual

time steps.<sup>2</sup> We present the probability of where rates will be at each time step in one percent “rate buckets.” The forecast for 3-month Treasury yields is shown in this graph:

SAS Institute Inc.  
Simulation Name HJM Simulation of U.S. Treasury Yield Curve  
Simulation Start Date: July 3, 2025  
Number of Scenarios: 75000

Distribution of Future Values of the 3-Month Treasury Bill Rate (Probability of Being within Range, Percent)

		Years to Maturity																			
		0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10
Lower Bound of T-bill Level (Percent)	20.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	19.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	18.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00
	17.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.01	0.01	0.01	0.00
	16.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
	15.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.02	0.03	0.03	0.02	0.02	0.04	0.04	0.03
	14.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.05	0.04	0.06	0.04	0.05	0.08	0.07	0.06	0.08	0.09
	13.00	0.00	0.00	0.00	0.00	0.00	0.02	0.01	0.03	0.05	0.05	0.09	0.10	0.09	0.13	0.12	0.11	0.16	0.15	0.17	0.20
	12.00	0.00	0.00	0.00	0.00	0.01	0.03	0.07	0.07	0.09	0.15	0.15	0.21	0.21	0.23	0.23	0.25	0.28	0.27	0.31	0.31
	11.00	0.00	0.00	0.00	0.01	0.02	0.08	0.11	0.14	0.17	0.24	0.28	0.31	0.39	0.40	0.46	0.49	0.43	0.52	0.50	0.53
10.00	0.00	0.00	0.01	0.02	0.11	0.13	0.20	0.28	0.35	0.43	0.55	0.54	0.65	0.76	0.70	0.68	0.77	0.75	0.83	0.86	
9.00	0.00	0.01	0.03	0.11	0.21	0.33	0.43	0.51	0.59	0.72	0.84	0.96	0.99	1.05	1.13	1.16	1.13	1.22	1.19	1.29	
8.00	0.00	0.07	0.13	0.23	0.47	0.66	0.81	0.86	1.04	1.20	1.36	1.52	1.54	1.50	1.64	1.64	1.59	1.71	1.73	1.68	
7.00	0.01	0.37	0.46	0.62	1.04	1.31	1.43	1.64	1.76	1.85	2.07	2.10	2.30	2.23	2.30	2.39	2.38	2.43	2.41	2.58	
6.00	0.22	1.79	1.42	1.65	2.22	2.54	2.53	2.59	2.90	2.98	3.10	3.24	3.41	3.28	3.24	3.18	3.25	3.19	3.33	3.30	
5.00	3.06	6.56	4.14	4.05	4.76	4.82	4.65	4.64	4.61	4.72	4.88	4.99	4.74	4.79	4.46	4.54	4.52	4.55	4.60	4.61	
4.00	20.12	18.06	10.20	8.89	9.05	8.61	7.96	7.68	7.32	7.45	7.27	7.12	7.02	7.12	6.84	6.65	6.66	6.55	6.48	6.56	
3.00	45.82	31.71	20.75	16.83	15.61	14.62	13.57	12.48	12.20	11.75	11.21	11.06	10.81	10.33	10.34	10.04	10.00	9.61	9.94	9.67	
2.00	27.71	29.73	29.47	25.57	23.38	21.41	20.15	18.90	18.10	17.53	17.23	16.72	16.25	16.16	15.79	15.59	15.36	15.45	14.89	14.75	
1.00	3.04	10.65	24.02	26.45	25.45	24.73	24.50	24.44	23.72	23.27	22.95	22.70	22.43	22.14	22.27	22.29	22.06	22.01	21.86	21.85	
0.00	0.03	1.04	8.57	13.43	14.68	16.69	18.33	19.48	19.98	20.02	20.10	20.35	20.63	20.87	21.23	21.46	21.59	21.65	21.78	21.72	
-1.00	0.00	0.01	0.78	2.06	2.83	3.74	4.77	5.61	6.26	6.59	6.70	6.84	7.22	7.58	7.65	7.91	8.11	8.17	8.13	8.20	
-2.00	0.00	0.00	0.01	0.09	0.15	0.27	0.44	0.60	0.78	0.90	1.05	1.04	1.10	1.18	1.34	1.34	1.38	1.44	1.50	1.52	
-3.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.04	0.06	0.08	0.10	0.12	0.11	0.14	0.15	0.16	0.18	0.18	0.21	0.21	
-4.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.02	0.03	0.02	0.02	0.01	

### 3-Month U.S. Treasury Yield Data Are Attached.

The probability that the 3-month Treasury bill yield will be between 1% and 2% in 2 years is shown in column 4: 26.45%. The probability that the 3-month Treasury bill yield will be negative (as it has been often in Europe and Japan) in 2 years is 2.06% plus 0.09% plus 0.00% = 2.14% (difference due to rounding). Cells shaded in blue represent positive probabilities of occurring, but the probability has been rounded to the nearest 0.01%. The shading scheme is defined as follows:

- Dark blue: the probability is greater than 0% but less than 1%
- Light blue: the probability is greater than or equal to 1% and less than 5%
- Light yellow: the probability is greater than or equal to 5% and 10%
- Medium yellow: the probability is greater than or equal to 10% and less than 20%
- Orange: the probability is greater than or equal to 20% and less than 25%
- Red: the probability is greater than 25%

The chart below shows the same probabilities for the 10-year U.S. Treasury yield derived as part of the same simulation.

<sup>2</sup> The full simulation uses 91-day time steps for 30 years forward. This note summarizes just the first 10 years of the full simulation.



SAS Institute Inc.  
Simulation Name: HJM Simulation of U.S. Treasury Yield Curve  
Simulation Start Date: July 3, 2025  
Number of Scenarios: 75000

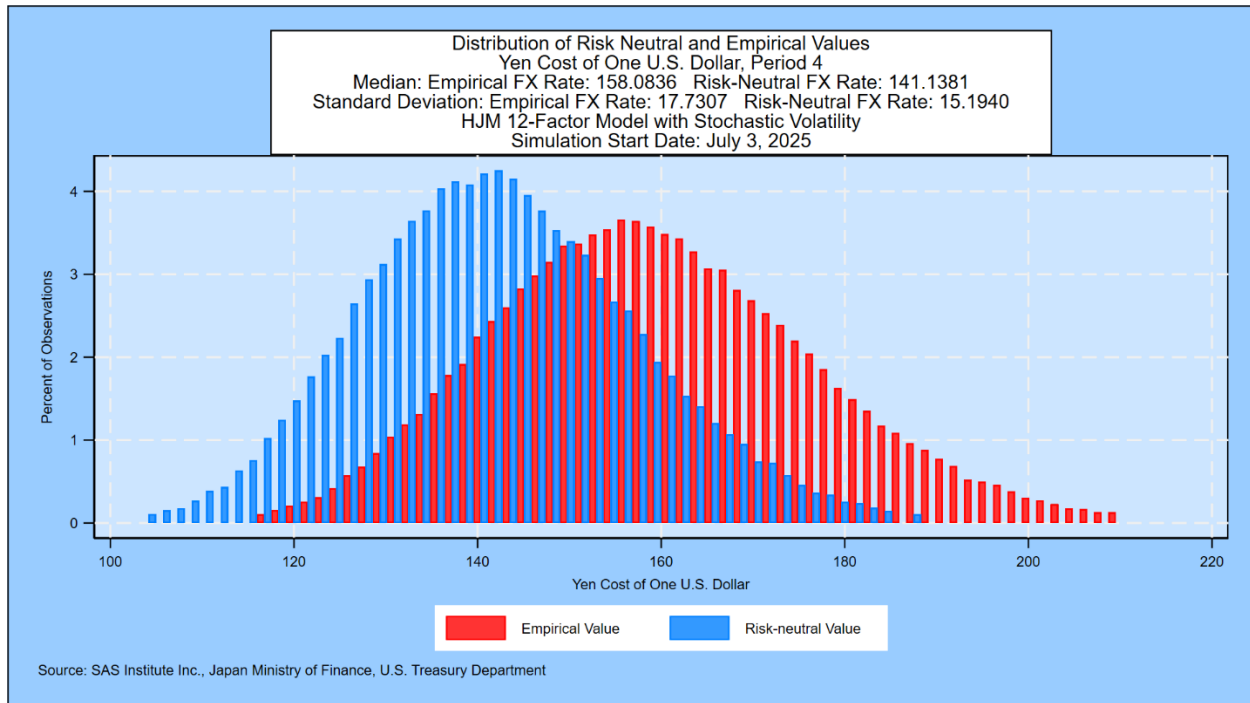
Distribution of Future Values of the 10-Year Treasury Bond Yield (Probability of Being within Range, Percent)

	Years to Maturity																			
	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10
Lower Bound of Yield Level (Percent)	17.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	16.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	15.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.01	0.00	0.01
	14.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02
	13.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.05	0.06	0.06
	12.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.04	0.06	0.06	0.08	0.10	0.12	0.11	0.14	0.12	0.13	0.13
	11.00	0.00	0.00	0.00	0.01	0.01	0.03	0.06	0.10	0.13	0.17	0.19	0.22	0.24	0.25	0.26	0.29	0.30	0.29	0.31
	10.00	0.00	0.00	0.00	0.01	0.03	0.08	0.13	0.22	0.33	0.33	0.39	0.43	0.47	0.51	0.52	0.58	0.56	0.68	0.67
	9.00	0.00	0.00	0.02	0.07	0.17	0.29	0.40	0.54	0.60	0.77	0.84	0.93	0.98	1.08	1.16	1.16	1.23	1.18	1.25
	8.00	0.00	0.01	0.13	0.35	0.62	0.84	1.15	1.31	1.48	1.64	1.79	1.88	2.01	2.11	2.17	2.31	2.27	2.25	2.24
	7.00	0.01	0.27	0.84	1.35	2.03	2.40	2.65	2.96	3.22	3.41	3.56	3.73	3.80	3.78	3.81	3.84	3.98	3.95	3.89
	6.00	0.47	2.52	3.95	4.93	5.40	5.90	6.12	6.37	6.38	6.63	6.62	6.59	6.48	6.58	6.54	6.43	6.42	6.38	6.32
	5.00	10.67	13.32	13.02	12.95	12.91	12.59	12.23	11.79	11.63	11.32	11.02	10.79	10.67	10.37	10.32	10.18	10.12	10.01	9.78
	4.00	46.23	33.63	28.41	25.06	22.53	20.87	19.96	18.94	18.17	17.30	17.03	16.52	16.15	15.71	15.26	14.87	14.57	14.47	14.48
	3.00	37.94	36.10	32.55	30.17	27.99	26.24	24.82	23.94	23.15	22.44	21.71	21.19	20.70	20.40	20.01	19.72	19.52	19.45	19.32
	2.00	4.66	12.95	17.65	19.28	20.29	20.88	20.92	21.18	21.05	21.06	21.07	21.02	21.04	21.07	21.23	21.28	21.37	21.15	21.21
	1.00	0.04	1.16	3.29	5.39	7.24	8.56	9.83	10.53	11.32	12.07	12.57	13.17	13.66	14.09	14.41	14.67	14.79	15.11	15.29
	0.00	0.00	0.03	0.14	0.43	0.79	1.28	1.67	2.05	2.41	2.70	2.99	3.25	3.50	3.69	3.94	4.15	4.41	4.53	4.62
	-1.00	0.00	0.00	0.00	0.00	0.01	0.04	0.07	0.09	0.12	0.14	0.15	0.19	0.20	0.22	0.22	0.26	0.27	0.31	0.31
	-2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00

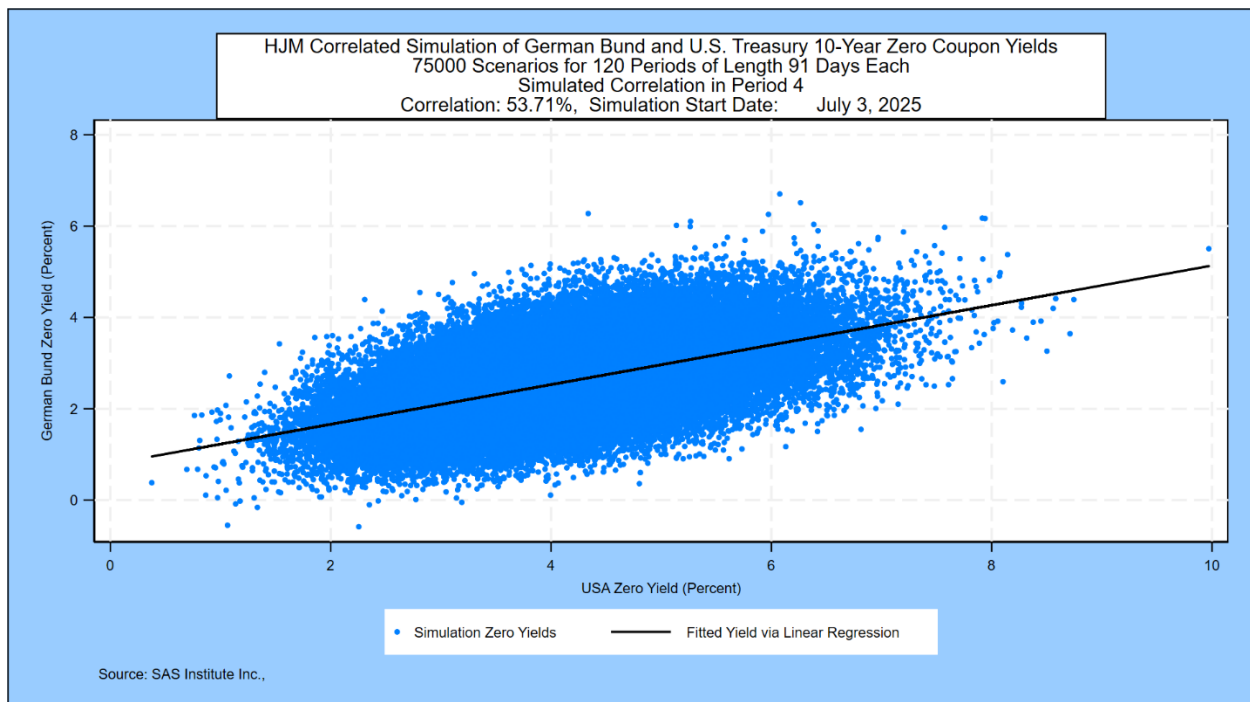
10-Year US Treasury Yield Data Are Attached.

## Modeling Correlated Foreign Exchange Rates and Government Yields

The SAS HJM++<sup>®</sup> methodology is based on five decades of research by Prof. Robert Jarrow. Using the no-arbitrage conditions of Heath, Jarrow and Morton [1992] and foreign exchange rate no-arbitrage conditions from Amin and Jarrow [1992], foreign exchange rates and government yields in major markets are simulated jointly. The simulation is based on daily movements in government yields and foreign exchange rates since January 2, 1962. The resulting distribution of the yen-U.S. dollar exchange rate one year forward is shown in this graph:



Similarly, the forward-looking correlation between 10-year zero-coupon Bund and U.S. Treasury yields is shown below:



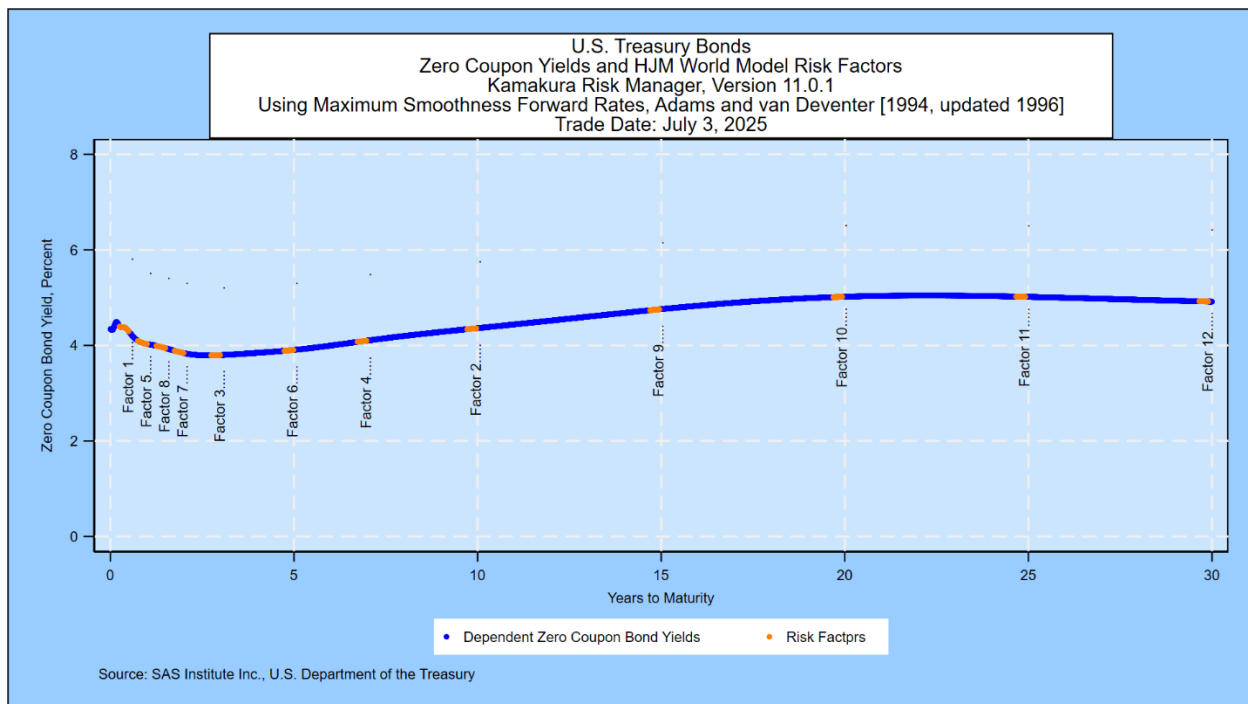
For more information on HJM++ scenario generation and no-arbitrage Monte Carlo simulation, please contact the author.

## Appendix: Treasury Simulation Methodology

The Treasury yield probabilities are derived using the same methodology that SAS Institute Inc. recommends to its KRIS® and Kamakura Risk Manager® clients. A moderately technical explanation is given later in the appendix, but we summarize it in plain English first.

Step 1: We take the [closing U.S. Treasury yield curve](#) as our starting point.

Step 2: We use the number of points on the yield curve that best explains historical yield curve shifts. Using daily government bond yield data from 14 countries from 1962 through May 31, 2025, we conclude that 12 “factors” drive almost all movements of government bond yields. The countries on which the analysis is based are Australia, Canada, France, Germany, Italy, Japan, New Zealand, Russia, Singapore, Spain, Sweden, Thailand, the United Kingdom, and the United States of America. No data from Russia is included after January, 2022. The graph below shows which points on the zero-coupon yield curve are incorporated as risk factors and the order in which they are added to the model:



Step 3: We measure the volatility of changes in those factors and how volatility has changed over the same period.

Step 4: Using those measured volatilities, we generate 75,000 random shocks at each time step and derive the resulting yield curve.

Step 5: We “validate” the model to make sure that the simulation EXACTLY prices the starting Treasury curve and that it fits history as well as possible. The methodology for doing this is described below.

Step 6: We take all 75,000 simulated yield curves and calculate the probabilities that yields fall in each of the 1% “buckets” displayed in the graph.

### **Do Treasury Yields Accurately Reflect Expected Future Inflation?**

We showed in a recent post on SeekingAlpha that, on average, investors have almost always done better by buying long term bonds than by rolling over short term Treasury bills. That means that market participants have generally (but not always) been accurate in forecasting future inflation and adding a risk premium to that forecast.

The distribution above helps investors estimate the probability of success from going long.

Finally, as mentioned weekly in the Corporate Bond Investor Friday overview, the future expenses (both the amount and the timing) that all investors are trying to cover with their investments are an important part of investment strategy. The author follows his own advice: cover the short-term cash needs first and then step out to cover more distant cash needs as savings and investment returns accumulate.

### **Technical Details**

Daily government bond yields from the 14 countries listed above form the base historical data for fitting the number of yield curve factors and their volatility. The U.S. historical data is provided by the [U.S. Department of the Treasury](#). The use of the international bond data increases the number of observations to more than 109,000 and provides a more complete range of experience with both high rates and negative rates than a U.S. data set alone provides.

The modeling process was published in [a very important paper](#) by David Heath, [Robert Jarrow](#) and Andrew Morton in 1992:

*Econometrica*, Vol. 60, No. 1 (January, 1992), 77–105

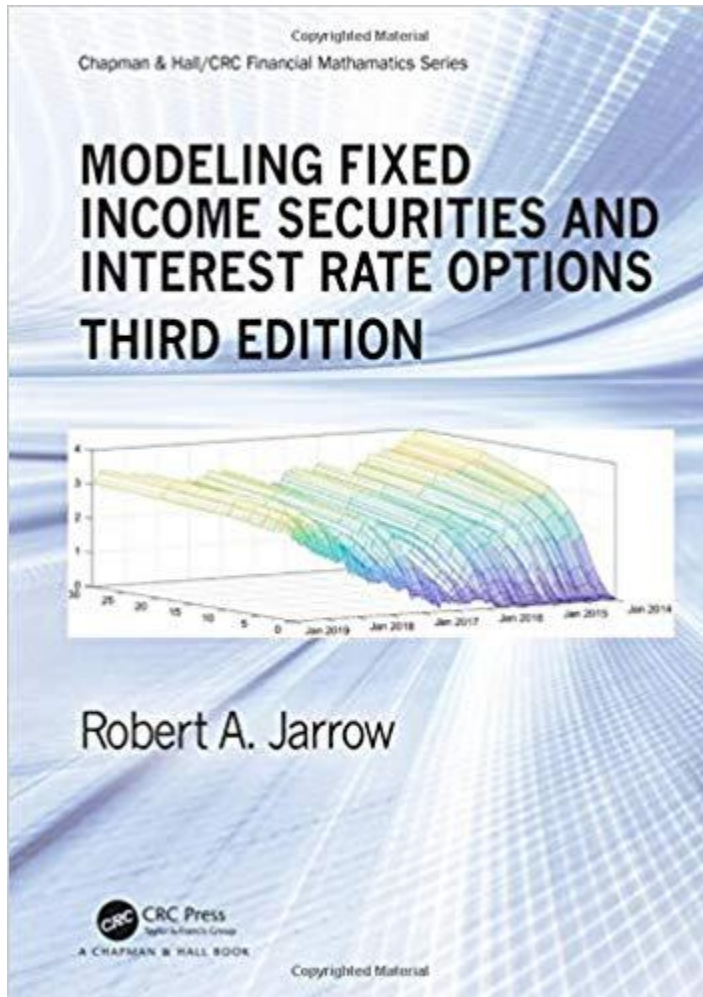
## **BOND PRICING AND THE TERM STRUCTURE OF INTEREST RATES: A NEW METHODOLOGY FOR CONTINGENT CLAIMS VALUATION<sup>1</sup>**

**BY DAVID HEATH, ROBERT JARROW, AND ANDREW MORTON<sup>2</sup>**

This paper presents a unifying theory for valuing contingent claims under a stochastic term structure of interest rates. The methodology, based on the equivalent martingale measure technique, takes as given an initial forward rate curve and a family of potential stochastic processes for its subsequent movements. A no arbitrage condition restricts this family of processes yielding valuation formulae for interest rate sensitive contingent claims which do not explicitly depend on the market prices of risk. Examples are provided to illustrate the key results.

Professor Jarrow's biography [is available here](#).

For technically inclined readers, we recommend Prof. Jarrow's book *Modeling Fixed Income Securities and Interest Rate Options* for those who want to know exactly how the "HJM" model construction works.



The number of factors, 12 for the 14-country model, has been stable since June 30, 2017. The volatilities for each factor are updated monthly and available to subscribers to the [KRIS interest rate and macro-factor data service](#).