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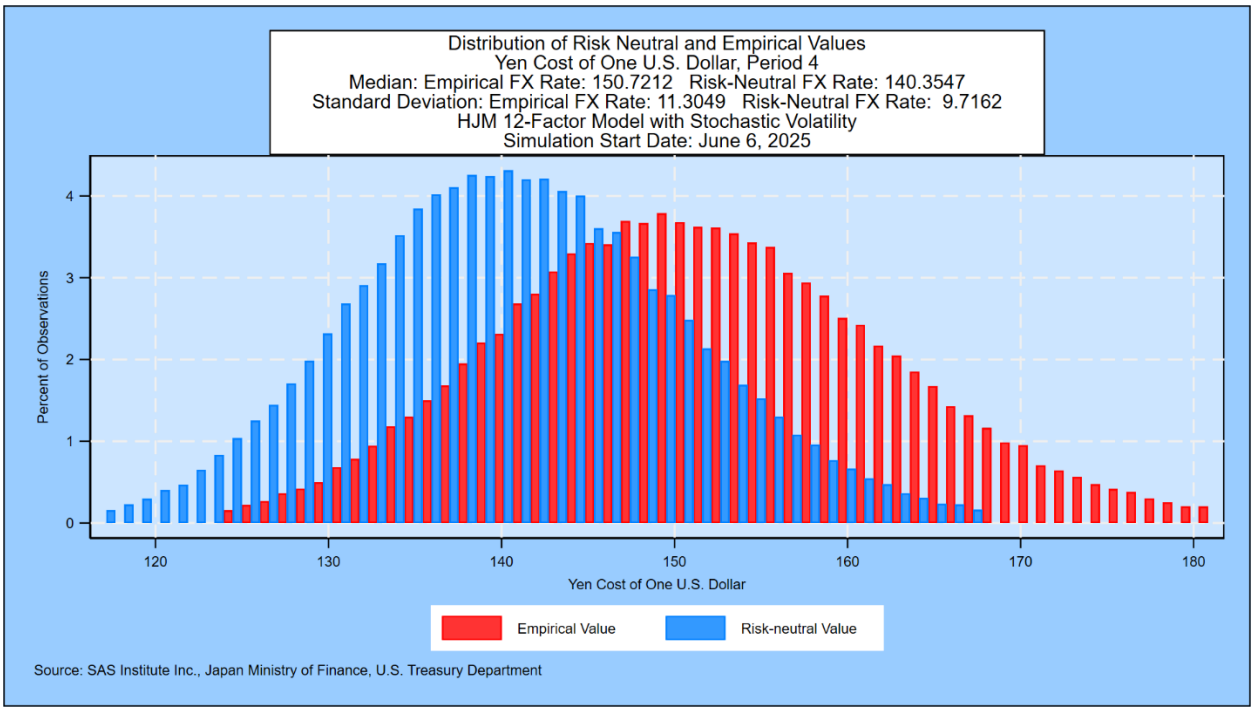
**SAS Weekly Japanese Government Bond and Yen Simulation, June 6, 2025: One-month Forward Bill Rate Peak at 5.86%, Down 0.31%**

## Summary

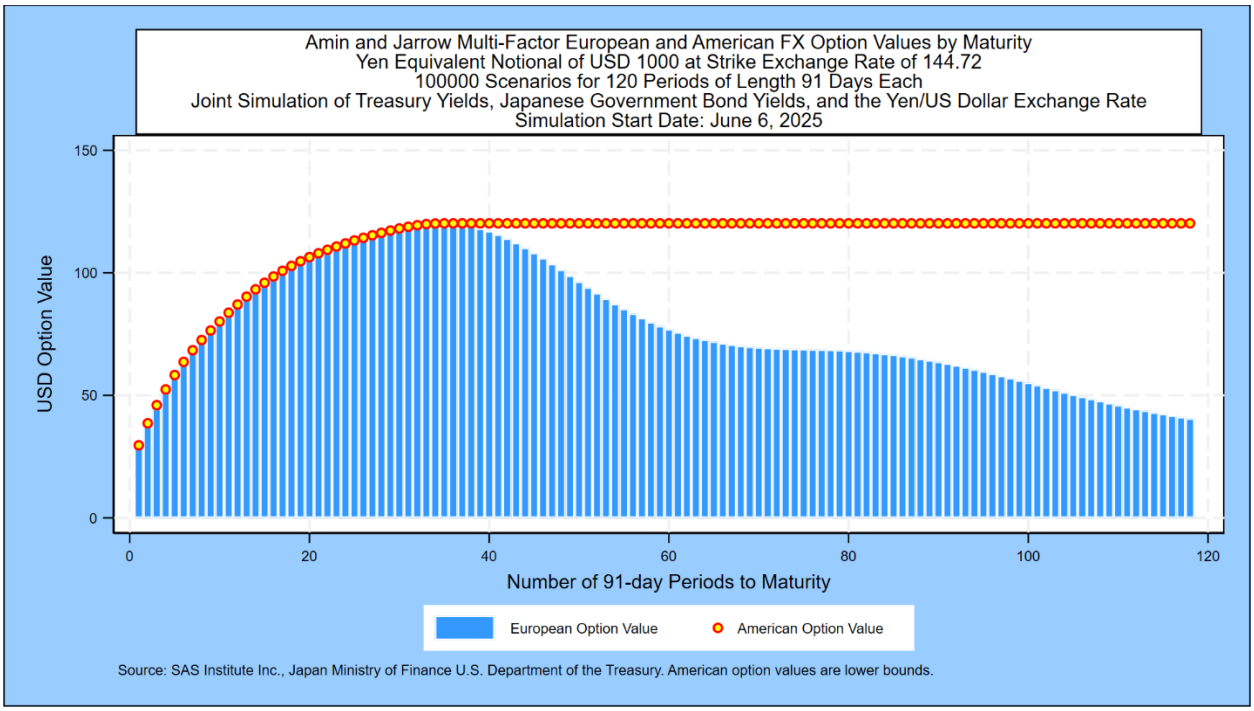

- The median level for the yen-U.S. dollar exchange rate is 150.72 one year from now, compared to 148.18 last week, according to this week's 100,000 scenario simulation of JGB yields and the exchange rate.
- The term premium in the JGB market maintains its "yield curve control" shape again this week, but forward rates on the long end have steepened considerably.
- The most likely one percent ranges for the 3-month yield (0% to 1%) and 10-year yield (2% to 3%) in 10 years are unchanged this week.
- The simulation with U.S. Treasuries shows a yen/U.S. dollar exchange rate with a standard deviation of 11.30 one year forward.
- The same simulation is used to price short and long-dated foreign exchange options on the yen versus the U.S. dollar at a strike price of 144.72 yen.


## Author's Note

*This simulation has been done jointly with a U.S. Treasury yield simulation in a way that reflects the correlation among the 12 factors driving yields in each country. For more on the companion U.S. Treasury simulation, please contact the author. In addition, foreign exchange rates include very substantial idiosyncratic risk that is independent of interest rate factors. Both this idiosyncratic risk and the JGB and the U.S. Treasury yield simulations impact foreign exchange rates, resulting in the following distribution of the yen/U.S. dollar exchange rate one year forward:*

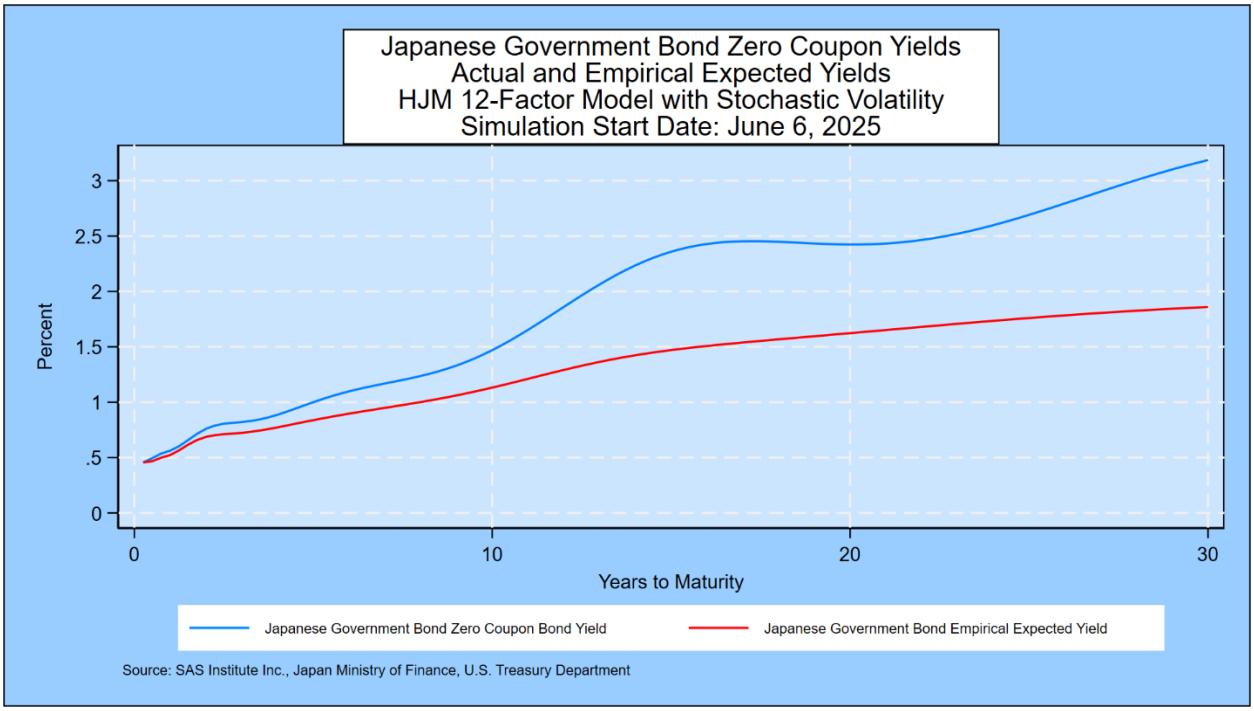



Pricing for short- and long-dated European options to buy yen versus U.S. dollars at a strike price of 144.72 for quarterly maturities out to 30 years is shown in the following graph. Note that the pricing for American options is the lower bound on fair-value pricing.




## This Week's Simulation of Japanese Government Bond Yields

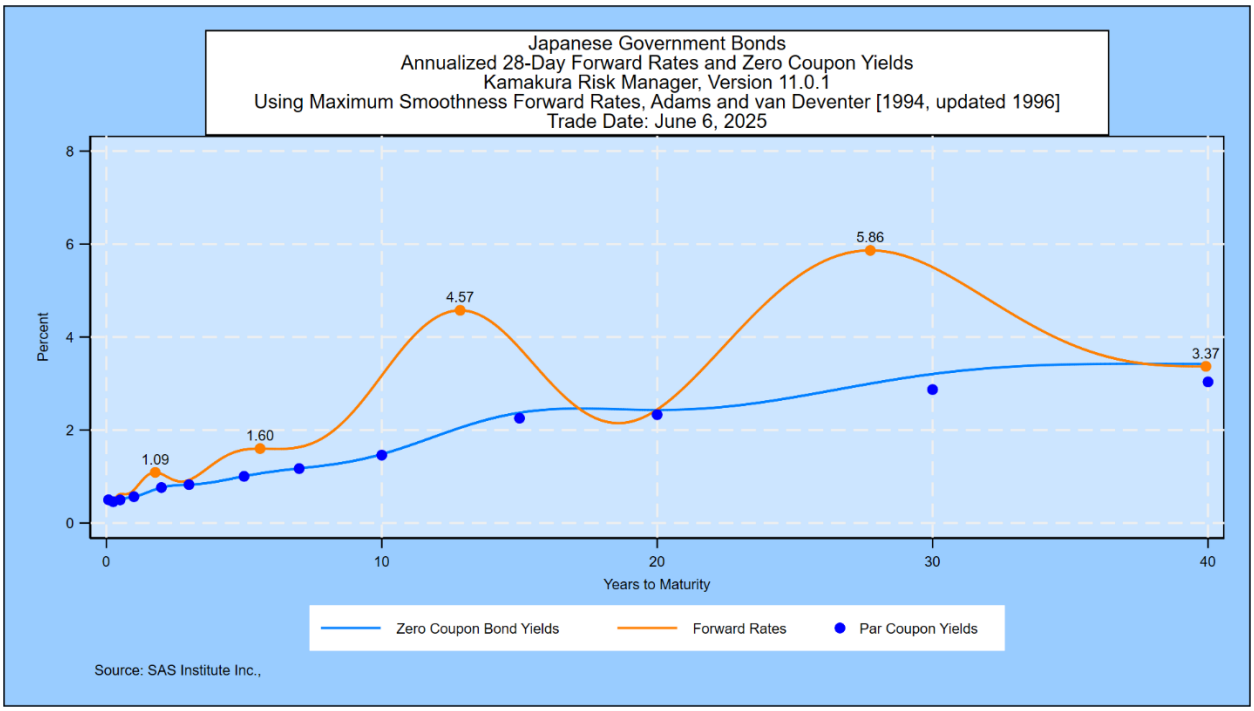
As explained in Prof. Robert Jarrow's book cited below, forward rates contain a risk premium above and beyond the market's expectations for the 3-month forward rate. We document the size of that risk premium in the graph below, which shows the zero-coupon yield curve implied by current JGB prices compared with the annualized compounded yield on 3-month bills that market participants would expect based on the daily movement of government bond yields in 14 countries since 1962. After many years of the Bank of Japan's "yield curve control" and a few weeks of "normality," yields under 10 years to maturity show a narrow term premium typical of the "YCC" era.




For more on this topic, see the analysis of government bond yields in 14 countries through May 31, 2025 given in the appendix.

**Inverted Yields, Negative Rates, and JGB Probabilities 10 Years Forward**

In this week's Japan forecast, the focus is on three elements of interest rate behavior: the future probability of the recession-predicting inverted yield curve, the probability of negative rates, and the probability distribution of JGB yields over the next decade. We start from the closing JGB yield curve published daily by the Japan Ministry of Finance and other information sources. Using a maximum smoothness forward rate approach, Friday's implied forward rate curve shows 1-month rates at an initial level of 0.50%, compared to 0.43% last week. As maturities lengthen, there is an initial peak in forward rates near 12 years and the a second peak at 5.86%, versus 6.17% last week. Rates then fall to 3.37% at the end of the 40-year horizon,

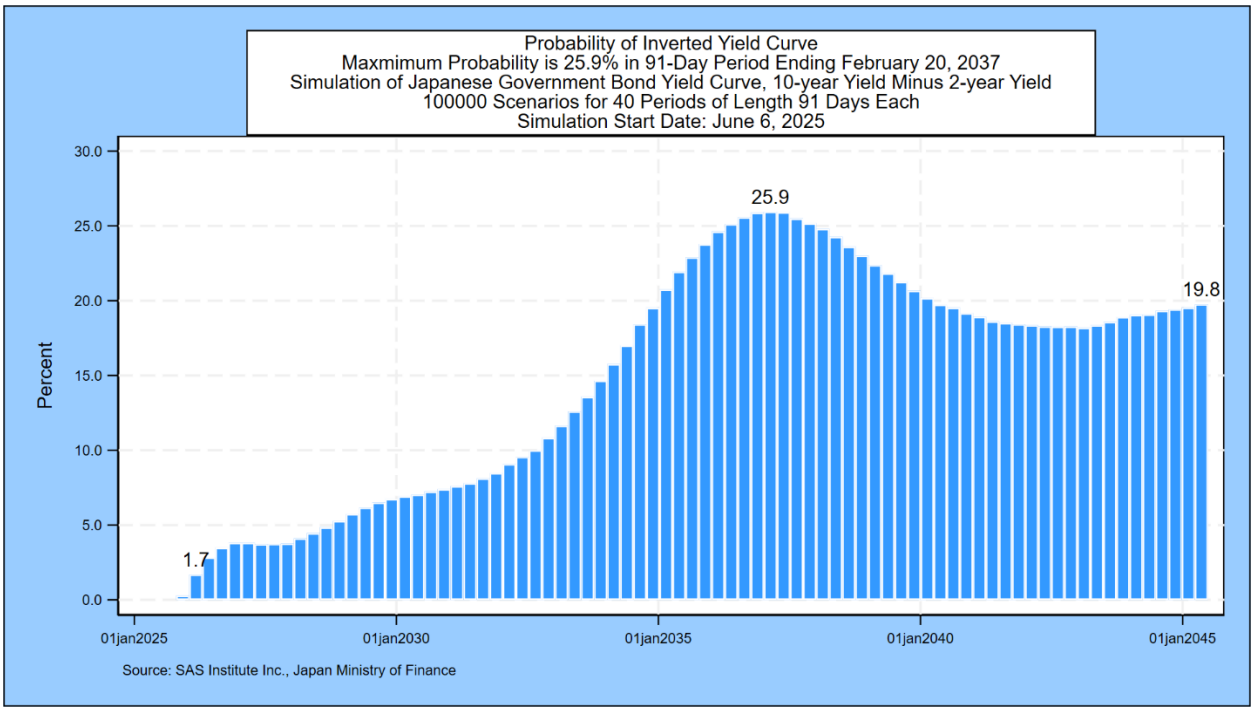



Using the methodology outlined in the appendix, we simulate 100,000 future paths for the JGB yield curve out to thirty years. The next three sections summarize our conclusions from that simulation.

**Inverted JGB Yields: 25.9% Probability by February 20, 2037**

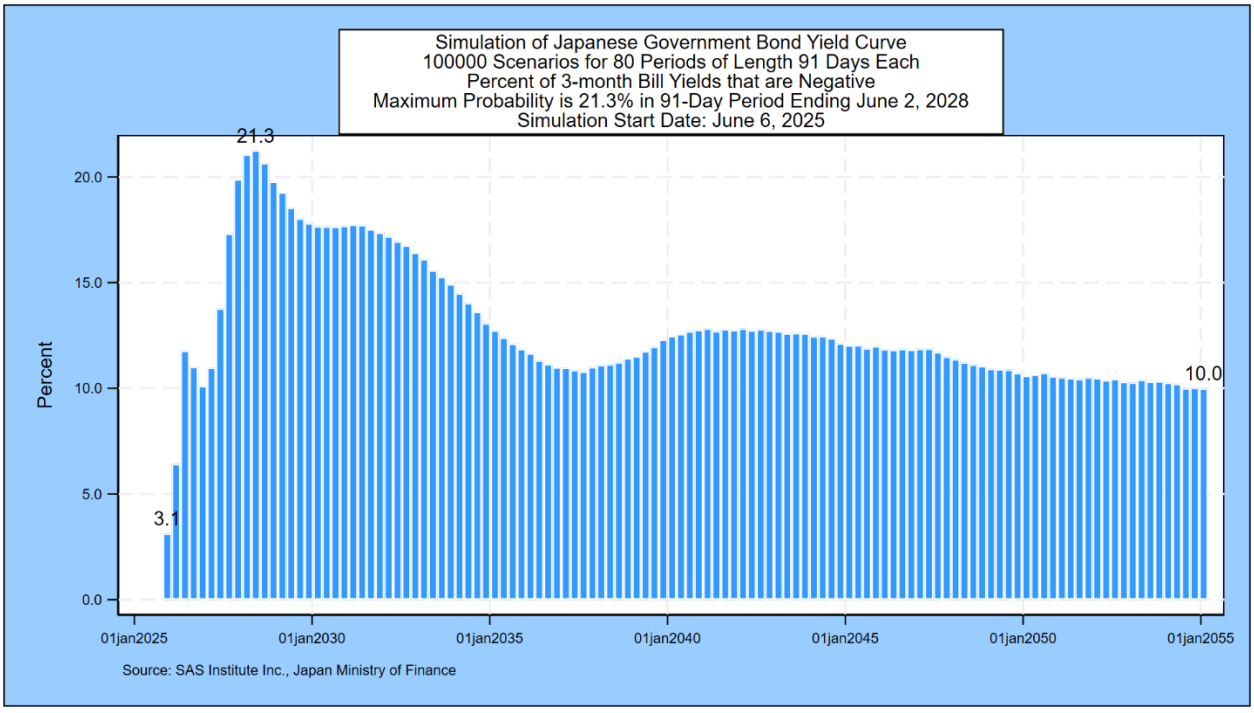
Many economists have concluded that a downward sloping yield curve is an important indicator of future recessions. A recent example is this paper by Alex Domash and Lawrence H. Summers. We measure the probability that the 10-year par coupon JGB yield is lower than the 2-year par coupon JGB for every scenario in each of the first 80 quarterly periods in the simulation.[1] The next graph shows that the probability of an inverted yield is near zero in the near term, but it peaks at 25.9%, compared to 27.3% last week, in the 91-day quarterly period ending February 20, 2037.

[1] After the first 20 years in the simulation, the 10-year yield cannot be derived from the initial 30-year term structure of yields.




**Negative 3-Month Yields: 21.3% Probability by June 2, 2028**

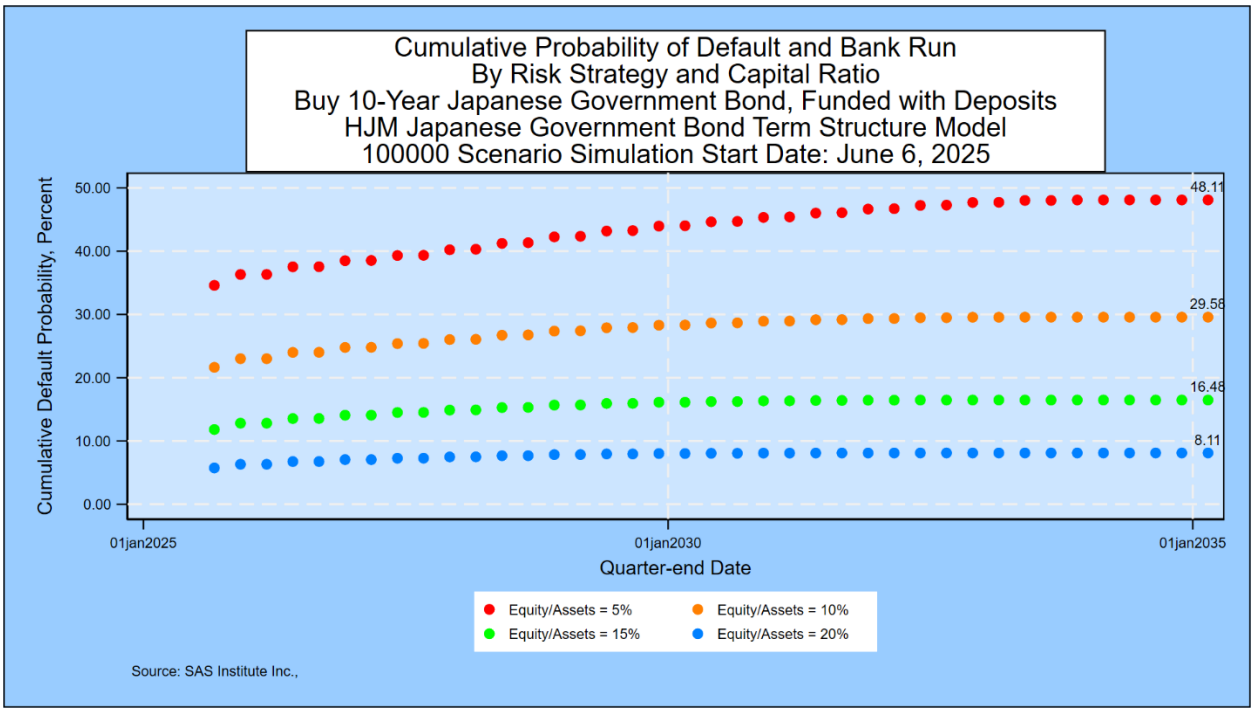
The next graph describes the probability of negative 3-month bill rates for all but the first 3 months of the next 3 decades. The probability of negative rates peaks at 21.3%, versus 19.3% last week, in the period ending June 2, 2028 and stays elevated at or above 10% thereafter.



## Calculating the Default Risk from Interest Rate Maturity Mismatches

In light of the interest-rate-risk-driven failure of Silicon Valley Bank in the United States on March 10, 2023, we have added a table that applies equally well to banks, institutional investor, and individual investor mismatches from buying long-term JGBs with borrowed short-term funds. We assume that the sole asset is a 10-year JGB purchased at time zero at par value of 100 yen. We analyze default risk for four different initial market values of equity to market value of asset ratios: 5%, 10%, 15%, and 20%. For the banking example, we assume that the only class of liabilities is deposits that can be withdrawn at par at any time. In the institutional and retail investor case, we assume that the liability is essentially a borrowing on margin/repurchase agreement with the possibility of margin calls. For all investors, the amount of liabilities (95, 90, 85 or 80) represents a “strike price” on a put option held by the liability holders. Failure occurs via a margin call, bank run, or regulatory takeover (in the banking case) when the value of assets falls below the value of liabilities.

The chart below shows the cumulative 10-year probabilities of failure for each of the 4 possible capital ratios when the asset’s maturity is 10 years. For the 5 percent case, that default probability is 48.11%, versus 48.41% in the prior week.




This default probability analysis is updated weekly based on the JGB yield simulation described in the next section. The calculation process is the same for any portfolio of assets with credit risk included.

## JGB Yield Probabilities 10 Years Forward

In this section, the focus turns to the decade ahead. This week's simulation shows that the most likely range for the 3-month bill yield in the JGB market in ten years is from 0% to 1%, unchanged from last week. There is a 26.41% probability that the 3-month yield falls in this range, compared to 26.22% one week before. Note that the probability of rates in the next-higher one percent range is also high. For the 10-year JGB yield, the most likely range is from 2% to 3%, also unchanged from last week. The probability of being in this range is 25.29%.

In a recent post on SeekingAlpha, we pointed out that a forecast of "heads" or "tails" in a coin flip leaves out critical information. What a sophisticated bettor needs to know is that, on average for a fair coin, the probability of heads is 50%. A forecast that the next coin flip will be "heads" is literally worth nothing to investors because the outcome is purely random.

The same is true for interest rates.

In this section we present the detailed probability distribution for both the 3-month bill rate and the 10-year JGB yield 10 years forward using semi-annual time steps[2]. We present the probability of where rates will be at each time step in one percent "rate buckets." The forecast for 3-month bill yields is shown in this graph:

**SAS Institute Inc.**
**Simulation Name** HJM Simulation of Japanese Government Bond Yield Curve
**Simulation Start Date:** June 6, 2025
**Number of Scenarios:** 100000

**Distribution of Future Values of the 3-Month Bill Rate (Probability of Being within Range, Percent)**

| Lower Bound of T-bill Level (Percent) | Years to Maturity 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 | 8 | 8.5 | 9 | 9.5 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 19.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 18.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 17.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 16.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 |
| 15.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.01 | 0.02 |
| 14.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.02 | 0.02 | 0.03 |
| 13.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.04 | 0.06 |
| 12.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.04 | 0.06 | 0.07 | 0.09 | 0.12 |
| 11.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.03 | 0.03 | 0.04 | 0.05 | 0.07 | 0.09 | 0.14 | 0.17 | 0.23 |
| 10.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.03 | 0.03 | 0.04 | 0.06 | 0.07 | 0.09 | 0.13 | 0.17 | 0.22 | 0.29 | 0.38 |
| 9.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.03 | 0.05 | 0.07 | 0.09 | 0.11 | 0.15 | 0.17 | 0.19 | 0.28 | 0.37 | 0.49 | 0.58 |
| 8.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.06 | 0.10 | 0.14 | 0.18 | 0.19 | 0.22 | 0.32 | 0.36 | 0.42 | 0.59 | 0.70 | 0.89 |
| 7.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.04 | 0.09 | 0.15 | 0.20 | 0.27 | 0.32 | 0.36 | 0.44 | 0.52 | 0.63 | 0.73 | 0.92 | 1.09 | 1.32 |
| 6.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.05 | 0.13 | 0.22 | 0.37 | 0.46 | 0.53 | 0.58 | 0.69 | 0.81 | 0.89 | 1.05 | 1.27 | 1.44 | 1.75 | 2.00 |
| 5.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.10 | 0.19 | 0.37 | 0.59 | 0.82 | 0.98 | 1.08 | 1.25 | 1.39 | 1.46 | 1.65 | 1.88 | 2.08 | 2.45 | 2.73 | 2.93 |
| 4.00 | 0.00 | 0.00 | 0.09 | 0.29 | 0.48 | 0.77 | 1.18 | 1.55 | 1.94 | 2.21 | 2.47 | 2.63 | 2.86 | 2.94 | 3.19 | 3.44 | 3.87 | 4.18 | 4.59 | 4.93 |
| 3.00 | 0.00 | 0.10 | 1.03 | 1.69 | 2.04 | 2.81 | 3.64 | 4.57 | 5.02 | 5.44 | 5.55 | 5.73 | 5.90 | 6.31 | 6.37 | 6.78 | 7.12 | 7.53 | 7.95 | 8.42 |
| 2.00 | 0.02 | 2.92 | 7.90 | 8.32 | 8.41 | 9.49 | 11.03 | 11.92 | 12.59 | 12.88 | 12.95 | 12.99 | 12.97 | 13.01 | 13.51 | 13.82 | 14.26 | 14.50 | 14.62 | 14.80 |
| 1.00 | 12.42 | 28.80 | 33.54 | 28.58 | 26.20 | 26.57 | 27.23 | 27.26 | 26.88 | 26.45 | 26.30 | 25.97 | 25.89 | 26.08 | 25.88 | 25.97 | 25.73 | 25.37 | 25.12 | 24.77 |
| 0.00 | 81.13 | 57.16 | 46.45 | 43.76 | 41.68 | 39.44 | 37.09 | 35.71 | 34.48 | 33.54 | 32.85 | 32.67 | 32.34 | 31.68 | 31.19 | 30.34 | 29.39 | 28.57 | 27.60 | 26.41 |
| -1.00 | 6.43 | 10.88 | 10.60 | 15.99 | 18.35 | 17.39 | 16.16 | 15.09 | 14.65 | 14.53 | 14.51 | 14.31 | 13.96 | 13.74 | 13.28 | 12.61 | 12.02 | 11.35 | 10.64 | 10.10 |
| -2.00 | 0.00 | 0.14 | 0.38 | 1.30 | 2.54 | 2.99 | 2.83 | 2.62 | 2.68 | 2.75 | 2.85 | 2.81 | 2.80 | 2.59 | 2.47 | 2.31 | 2.15 | 1.98 | 1.82 | 1.78 |
| -3.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.17 | 0.27 | 0.28 | 0.31 | 0.31 | 0.34 | 0.36 | 0.37 | 0.40 | 0.39 | 0.35 | 0.33 | 0.30 | 0.27 | 0.26 | 0.21 |
| -4.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.02 | 0.02 | 0.03 | 0.03 | 0.03 | 0.03 | 0.04 | 0.03 | 0.03 | 0.03 | 0.02 | 0.03 | 0.02 | 0.02 |
| -5.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| -6.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

## 3-Month Bill Yield Data:

[2] The actual simulation uses 91-day time steps and spans a 20-year time horizon.

The probability that the 3-month bill yield will be between 1% and 2% in 2 years is shown in column 4: 28.58%. The probability that the 3-month yield will be negative (as it has been often in Europe and Japan) in 2 years is 15.99% plus 1.30% plus 0.03% plus 0.00% = 17.32% (difference due to rounding). Cells shaded in blue represent positive probabilities of occurring, but the probability has been rounded to the nearest 0.01%. The shading scheme is defined as follows:

- Dark blue: the probability is greater than 0% but less than 1%
- Light blue: the probability is greater than or equal to 1% and less than 5%
- Light yellow: the probability is greater than or equal to 5% and 10%
- Medium yellow: the probability is greater than or equal to 10% and less than 20%
- Orange: the probability is greater than or equal to 20% and less than 25%
- Red: the probability is greater than 25%

The chart below shows the same probabilities for the 10-year JGB yield derived as part of the same simulation.

SAS Institute Inc.
Simulation Name HJM Simulation of Japanese Government Bond Yield Curve
Simulation Start Date: June 6, 2025
Number of Scenarios: 100000

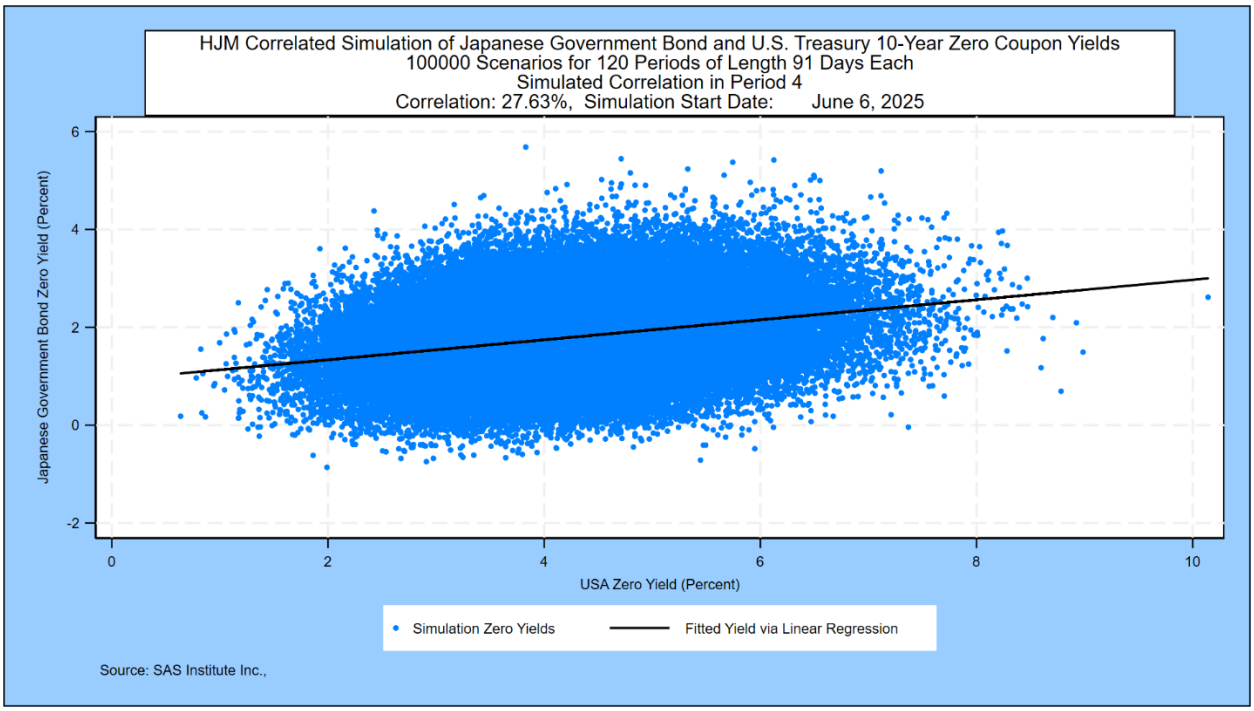
Distribution of Future Values of the 10-Year Japanese Government Bond Yield (Probability of Being within Range, Percent)

| Lower Bound of Yield Level (Percent) | Years to Maturity | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 | 8 | 8.5 | 9 | 9.5 | 10 |
| 15.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 14.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 |
| 13.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.00 | 0.01 |
| 12.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.04 | 0.04 |
| 11.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.02 | 0.02 | 0.03 | 0.05 | 0.05 | 0.08 | 0.09 | 0.08 | 0.09 |
| 10.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.06 | 0.09 | 0.10 | 0.13 | 0.16 | 0.21 | 0.20 | 0.25 |
| 9.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.02 | 0.04 | 0.06 | 0.09 | 0.13 | 0.17 | 0.23 | 0.28 | 0.33 | 0.40 | 0.44 | 0.51 | 0.57 |
| 8.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.08 | 0.12 | 0.20 | 0.28 | 0.37 | 0.43 | 0.57 | 0.68 | 0.74 | 0.86 | 0.95 | 1.02 | 1.07 |
| 7.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 0.06 | 0.14 | 0.27 | 0.41 | 0.58 | 0.77 | 0.96 | 1.19 | 1.26 | 1.48 | 1.69 | 1.72 | 1.84 | 2.01 | 2.10 |
| 6.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.14 | 0.33 | 0.61 | 0.93 | 1.32 | 1.70 | 2.02 | 2.33 | 2.61 | 2.96 | 3.15 | 3.33 | 3.55 | 3.57 | 3.67 | 3.76 |
| 5.00 | 0.00 | 0.01 | 0.09 | 0.35 | 0.88 | 1.56 | 2.35 | 3.13 | 3.89 | 4.38 | 4.93 | 5.29 | 5.64 | 5.87 | 6.13 | 6.26 | 6.53 | 6.72 | 6.70 | 6.79 |
| 4.00 | 0.00 | 0.19 | 1.12 | 2.68 | 4.34 | 6.09 | 7.50 | 8.68 | 9.58 | 10.32 | 10.59 | 10.90 | 11.17 | 11.36 | 11.42 | 11.51 | 11.58 | 11.80 | 11.63 | 11.75 |
| 3.00 | 0.45 | 4.34 | 8.93 | 12.68 | 15.37 | 17.29 | 18.63 | 19.26 | 19.58 | 19.71 | 19.85 | 19.70 | 19.54 | 19.39 | 19.32 | 19.24 | 19.09 | 18.91 | 19.04 | 18.70 |
| 2.00 | 21.32 | 30.64 | 33.06 | 33.09 | 33.14 | 32.22 | 31.24 | 30.39 | 29.35 | 28.69 | 27.97 | 27.76 | 27.10 | 26.64 | 26.27 | 26.08 | 25.69 | 25.35 | 25.37 | 25.29 |
| 1.00 | 65.82 | 50.43 | 42.49 | 37.43 | 33.47 | 30.75 | 28.59 | 26.95 | 25.95 | 24.93 | 24.33 | 23.69 | 23.47 | 23.19 | 22.77 | 22.33 | 22.22 | 21.89 | 21.65 | 21.55 |
| 0.00 | 12.35 | 14.02 | 13.65 | 12.84 | 11.74 | 10.76 | 10.04 | 9.49 | 8.98 | 8.64 | 8.45 | 8.15 | 7.97 | 7.78 | 7.75 | 7.70 | 7.56 | 7.62 | 7.45 | 7.43 |
| -1.00 | 0.07 | 0.37 | 0.66 | 0.87 | 0.90 | 0.93 | 0.85 | 0.79 | 0.77 | 0.76 | 0.68 | 0.66 | 0.61 | 0.61 | 0.57 | 0.57 | 0.54 | 0.58 | 0.60 | 0.59 |
| -2.00 | 0.00 | 0.00 | 0.01 | 0.01 | 0.01 | 0.02 | 0.01 | 0.02 | 0.01 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| -3.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

## 10-Year JGB Yield Data:

## Correlation with Multinational Government Yields

The Japanese government bond yield curve was simulated jointly with the U.S. Treasury yields and other government yields based on daily data since 1974. As a result, movements in JGB yields have a correlation with other government yields that is consistent with historical yield curve movements. The following graph shows the forward-looking correlation between 10-year JGB and U.S. Treasury zero coupon bond yields at the one-year point in the simulation:

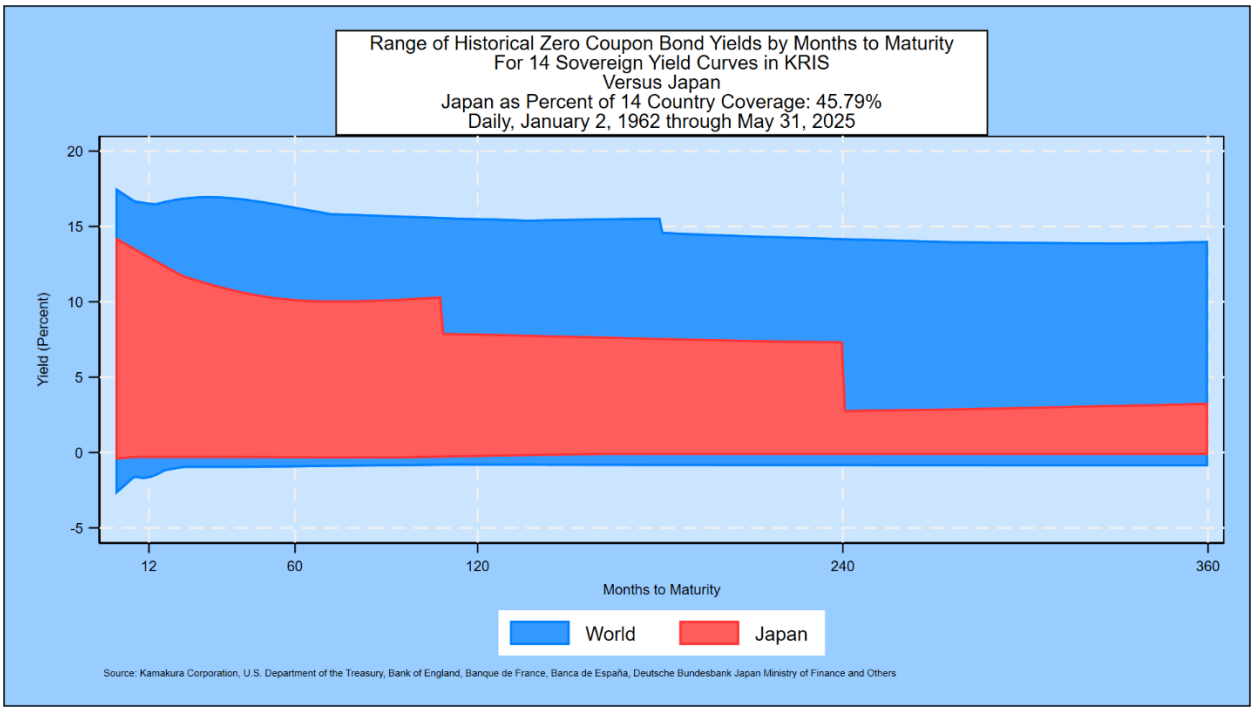



## Appendix: JGB Yield Simulation Methodology

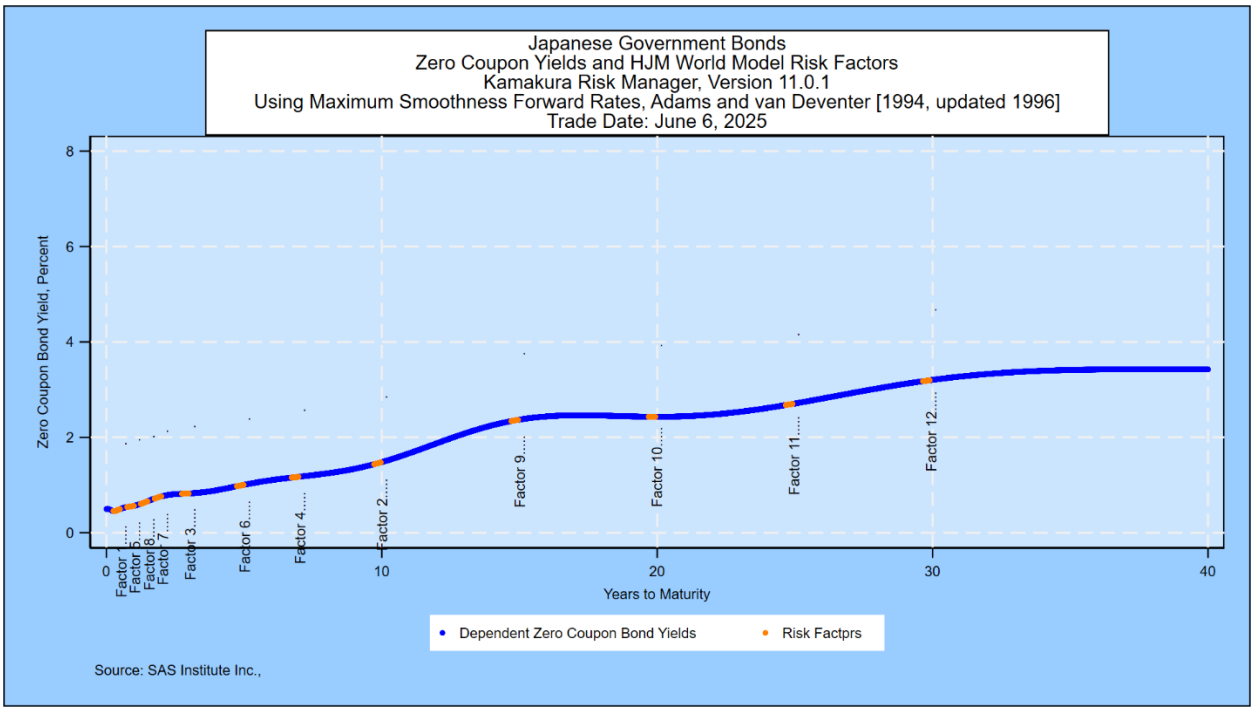
The probabilities are derived using the same methodology that SAS Institute Inc. recommends to its KRIS® and Kamakura Risk Manager® clients. A moderately technical explanation is given later in the appendix, but we summarize it briefly first.

Step 1: We take the closing JGB yield curve as our starting point.

Step 2: We use the number of points on the yield curve that best explains historical yield curve shifts. We note in the following graph that JGB yields span (by rate level and maturity) only 45.79% of the historical experience in 14 countries:




For the highest degree of realism in a forward-looking simulation, using the international database is essential. Using daily government bond yield data from 14 countries from 1962 through May 31, 2025, we conclude that 12 “factors” drive almost all movements of government bond yields. The countries on which the analysis is based are Australia, Canada, France, Germany, Italy, Japan, New Zealand. Russia, Singapore, Spain, Sweden, Thailand, the United Kingdom, and the United States of America. No data from Russia is included after January 2022. The factors and the order in which they are added to the model are shown in this plot of the current JGB zero-coupon yield curve:




Step 3: We measure the volatility of changes in those factors and how volatility has changed over the same period.

Step 4: Using those measured volatilities, we generate 100,000 random shocks at each time step and derive the resulting yield curve.

Step 5: We “validate” the model to make sure that the simulation EXACTLY prices the starting JGB curve and that it fits history as well as possible. The methodology for doing this is described below.

Step 6: We take all 100,000 simulated yield curves and calculate the probabilities that yields fall in each of the 1% “buckets” displayed in the graph.

**Do Nominal Yields Accurately Reflect Expected Future Inflation?**

We showed in a recent post on SeekingAlpha that, on average, investors have almost always done better by buying long term bonds than by rolling over short term Treasury bills in the United States. That means that market participants have generally (but not always) been accurate in forecasting future inflation and adding a risk premium to that forecast. This study is being updated using the 14-country data set in coming weeks.

**Technical Details**

Daily government bond yields from the 14 countries listed above form the base historical data for fitting the number of yield curve factors and their volatility. The JGB historical data is provided by the Japan Ministry of Finance. The use of the international bond data

increases the number of observations to more than 109,000 and provides a more complete range of experience with both high rates and negative rates than a JGB data set alone provides.

The modeling process was published in a very important paper by David Heath, Robert Jarrow and Andrew Morton in 1992:

*Econometrica*, Vol. 60, No. 1 (January, 1992), 77–105

# BOND PRICING AND THE TERM STRUCTURE OF INTEREST RATES: A NEW METHODOLOGY FOR CONTINGENT CLAIMS VALUATION[1]

BY DAVID HEATH, ROBERT JARROW, AND ANDREW MORTON[2]

This paper presents a unifying theory for valuing contingent claims under a stochastic term structure of interest rates. The methodology, based on the equivalent martingale measure technique, takes as given an initial forward rate curve and a family of potential stochastic processes for its subsequent movements. A no arbitrage condition restricts this family of processes yielding valuation formulae for interest rate sensitive contingent claims which do not explicitly depend on the market prices of risk. Examples are provided to illustrate the key results.

Professor Jarrow's biography is available here.

The no-arbitrage foreign exchange rate simulation is based on this well-known paper by Amin and Jarrow:

Journal of International Money and Finance
Volume 10, Issue 3, September 1991, Pages 310-329

# Pricing foreign currency options under stochastic interest rates

Kaushik I. Amin, Robert A. Jarrow

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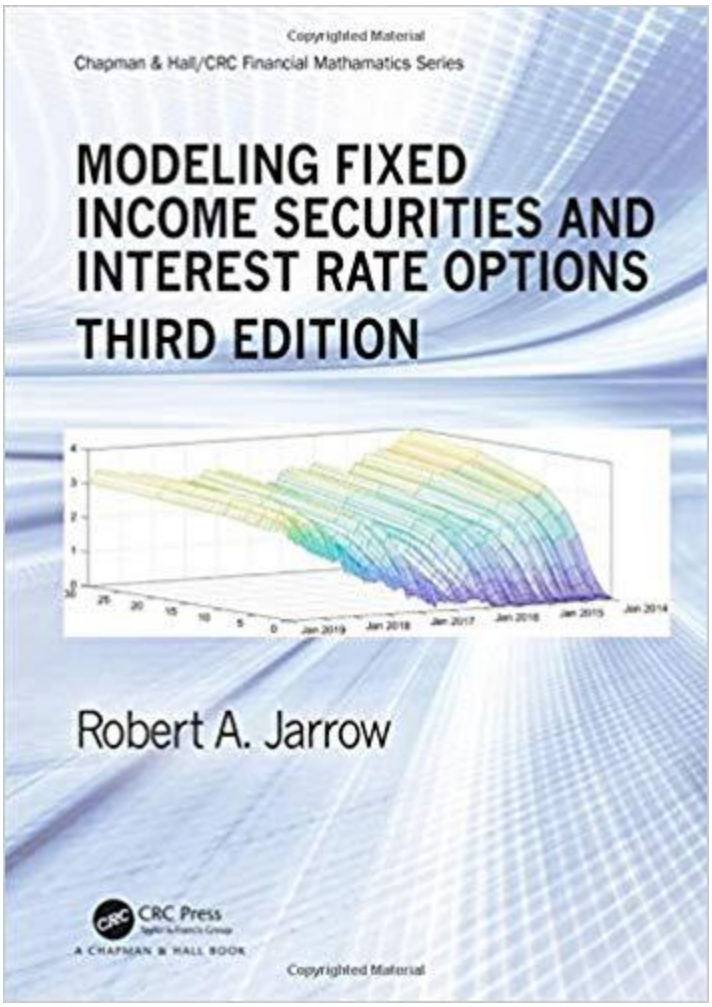
https://doi.org/10.1016/0261-5606(91)90013-A



## Abstract

In this paper, we build a general framework to price contingent claims on foreign currencies using the Heath *et al.* (1987) model of the term structure. Closed form solutions are obtained for European options on currencies and currency futures assuming that the volatility functions determining the term structure are deterministic. As such, this paper provides an example of a bond price process (for both the domestic and foreign economies) consistent with Grabbe's (1983) formulation of the same problem.

For technically inclined readers, we recommend Prof. Jarrow's book *Modeling Fixed Income Securities and Interest Rate Options* for those who want to know exactly how the "HJM" model construction works.




The number of factors, 12 for the 14-country model, has been stable since June 30, 2017.

**Footnotes:**