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Title: Designing Couercyclical and Risk Based Aggregate Deposit Insurance Premia

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Designing Countercyclical and Risk Based Aggregate Deposit Insurance Premia*

Abstract

This paper proposes a countercyclical and risk based aggregate deposit insurance premium design where the system attains a given survival probability over a fixed horizon. The fixed horizon is determined by economic and political considerations. Such a premium system necessarily exceeds actuarial fair value and results in the insurance fund growing over time. To mitigate this growth, the proposed system includes a swap contract that reduces the premia when the fund size exceeds a threshold. The system is made countercyclical by including another swap contract that exchanges premia in good times for relief in bad times. The costs for obtaining such a countercyclical deposit insurance premium system are documented.

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1 Introduction

The Federal Deposit Insurance Reform Act of 2005 permits the FDIC to charge every bank a premium based on risk, provide initial assessment credits to banks that helped to build up the insurance funds, and require the FDIC to pay rebates if the ratio of insurance fund size to insured deposits (reserve ratio) exceeds certain thresholds. While theoretically very appealing, concerns have been raised about the procyclical adverse impact of risk based premia and capital requirements: regulations require banking organizations to pay higher premia in economic downturns, thereby aggravating the effects of economic recessions (Blinder and Wescott, 2001). Pennacchi (1999) provides evidence that during recessions banks reduce deposits to mitigate the effects of higher premiums, which in turn reduces bank credit.

With the exception of Pennacchi (2006) not much research exists that proposes methods to mitigate the effects of the cyclical movements in deposit insurance premiums.¹ Pennacchi shows that one way to minimize the adverse effects of the cyclical premiums is to set the fair insurance rates as a moving average of future losses. We add to this scant research and propose an aggregate premium policy that is countercyclical by design and is nonetheless founded on risk based principles.

The basic idea is to build into the premium system a mechanism for trading higher premia in good times for relief in bad times. In addition, the proposed design may also limit the growth in the size of the deposit insurance fund that could occur for example, in a prolonged economic boom. This feature can be engineered by incorporating a premium reduction when the fund size exceeds a target level. The design therefore builds into the aggregate based premium policy an elasticity for both downturns and fund growth, with premiums taking a percentage reduction in response to the depth of the downturn and the excess fund size.

In addition to meeting these objectives with respect to recessionary relief and control over fund

¹ The countercyclical effects of capital regulation has attracted a wider reserach interest. Kashyap and Stein (2004), Allen and Saunders (2004), Gordy and Howells (2004), and White (2006) provide a review of these studies.

size growth, the premium system proposed is designed to be risk based. The premiums are thereby necessarily sensitive to the risk exposure of the system. Such an objective is easily met by ensuring that the premium system meets a prespecified risk target. For example, one may require that the risk level, as measured by a variety of risk measures, be kept below a target level. The resulting sensitivity of premia to risk is assured as any natural increases in risk exposures will then require enhanced premia to maintain the target risk level. Among the simpler and popular risk measures are the expected shortfall or the expected loss in some lower quantile and value-at-risk that merely controls the probability of a large loss. The latter risk target is more robust and here we illustrate with a value-at-risk type risk measure. Alternative measures may easily be incorporated.

In this study the target fund size, aggregate premium level and rebate structures are all kept risk based by ensuring that the deposit insurance system has a high probability of survival over a fixed horizon. The fixed horizon would be determined by political and economic considerations. For the purposes of our investigation, we select a 10-year horizon. We first establish a benchmark case where there are no premium rebates. In such a system we determine the target fund size and aggregate premium level that ensures fund survival over a 10-year horizon. We then evaluate the effects of altering rebates and premium levels and fund-size targets in the neighborhood of the benchmark level on the 10-year survival probability. We report the trade-offs that are implicit between these policies when we enforce a high 10-year survival probability.

We determine the 10-year survival probability by simulating the system through time. To determine the basic trade-offs, we consider a time homogeneous formulation of the risks involved and we perform all calculations in a steady state economy where deposit growth matches the spot rate of interest. The insurance system risks are the time series of losses paid out. We build a model for the aggregate loss distribution that entails three uncertain components. These components are the number of loss events (bank failures), the asset size of the failing banks, and the loss rate given default.

The model employs the unconditional distributions of these components. We model the number

of bank failures by a Poisson process with a constant arrival rate. For the distribution of asset sizes, we analyze the relevant data for US banks over the years 2000-2003. We confirm that a Frechet distribution provides a good statistical fit. With respect to loss rates, we analyze the loss rates experienced by the Federal Deposit Insurance Corporation over the period 1984-2002 on 1508 bank failures. We show that the Weibull model provides a good statistical fit to the historical loss experience.

Using these loss components we simulate the effects of the alternative risk based and counter-cyclical premium policies on the 10-year survival probability. The final results demonstrate the trade-off between the various policy dimensions on the 10-year survival probability. The specific policy dimensions investigated are the downturn rebate, the rebate for excessive growth of the fund, the level of aggregate premiums, and the target fund size.

We show that the cyclicity in premiums can be diminished, but at a cost. For example, in the simulation, we show that our loss and capital rebate system increases the aggregate premium approximately \$0.9 billion on average over ten-years to mitigate the impact of premium cyclicity. This additional premium represents a 35% increase in the \$2.6 billion premium paid for \$3.3 trillion of aggregate insured deposits.

The outline of the paper is as follows. Section 2 discusses the issues involved in risk based pricing of the aggregate deposit premium. Section 3 presents the design of the countercyclical and risk based premium system. The modeling results on asset sizes and loss rates are contained in Section 4. Section 5 describes the simulation of the 10-year survival probability, establishes the benchmarks for fund size and aggregate premiums, and constructs the trade-off table. Section 6 concludes.

2 Risk Based Pricing of the Aggregate Premium

We price the FDIC insurance premiums in aggregate, rather than pricing each individual's bank premium and summing across all banks. There is considerable research on deposit insurance pricing at the individual bank level.² To the best of our knowledge the model proposed herein is the first to estimate the FDIC's risk exposure at the aggregate level.

2.1 Complete Markets with Annual Contracting

If we assume that the market for bank failure losses is liquid, arbitrage free, and complete, then there exists a unique risk neutral measure Q , identifiable from the prices of traded securities, that can be used for valuation. Let $\tilde{E}_t(\cdot)$ denote conditional expectation under the risk neutral measure Q and let $E_t(\cdot)$ be the corresponding expectation operator under the statistical probability measure P .

For the analysis of the risks involved we adopt a discrete time model with annual periods denoted by $n = 0, \dots, N$. Let L_n be the level of losses, in billions of dollars, paid out by the FDIC Insurance fund in period n . The losses occur at the end of the period. Let κ_n be the aggregate premium, in billions of dollars per year, paid at the beginning of the period. Let r_n be the spot rate of interest for period n (from the beginning to the end of the period). The spot rate for time n is known at time n , but future spot rates ($t > n$) are random variables when viewed from time n . Then, complete-markets risk-neutral valuation implies that the fair insurance premium is:

$$\kappa_n = \frac{\tilde{E}_n(L_n)}{1 + r_n}. \quad (1)$$

If one believes that bank failure risk is diversifiable, then one should set annual premiums at their actuarially fair level or equal to the statistically expected loss level, in which case the risk neutral measure is replaced by the statistical measure in the previous expression. More generally,

² Merton, Markus and Schaked, Ronn and Verma, Pennacchi (1999) and Kuritzkes, Schuermann, and Weiner (2004), Duffie, Jarrow, Purnannandam, and Yang (2003).

for the pricing of options on L_n under the statistical measure, one needs a stronger form of diversifiability (see Atlan, Geman, Madan and Yor, 2006).

We observe, however, that if the statistical measure is used for determining premia then the premium system is not risk based. The reason is that movements in the distribution of losses with the same mean will result in the same deposit premia. Of course if these movements are diversifiable, then the market would charge no risk adjustment. However, this is unlikely and one would expect the deposit insurance premium to be sensitive to such risk exposures.

The premium κ_n , as determined by equation (1) is time varying and is based on the fluctuations of both market conditions (as reflected in the information set and the risk premia reflected in the change of probability embedded in the risk neutral conditional expectation operator) and the spot rate of interest. For various reasons, the banking industry may desire a constant premium $\bar{\kappa}$ set for a fixed period of time, say T years. The FDIC could enter into a T period cash flow swap (directly or synthetically) where it pays out the constant premium $\bar{\kappa}$ received, and it receives the fair premium. The constant premium $\bar{\kappa}$ is determined so that the swap written at time 0 occurs with no initial cash transfers, i.e. at zero value. Standard methods show that the premium $\bar{\kappa}$ should be set such that

$$\bar{\kappa} \sum_{n=0}^{T-1} B(0, n) = \sum_{n=0}^{T-1} \tilde{E} \left(\frac{\kappa_n(1+r_n)}{b_n} \right) \quad (2)$$

where $B(0, n) = \tilde{E}_0 \left(\frac{1}{b_n} \right)$ is the price at time 0 of a sure dollar paid at time n and $b_n = (1+r_0) \cdots (1+r_{n-1})$ is the time t value of a money market account growing at the spot rate of interest.

Using expression (1), this can equivalently be written as:

$$\bar{\kappa} = \frac{\sum_{n=0}^{T-1} \tilde{E} \left(\frac{L_n}{b_n} \right)}{\sum_{n=0}^{T-1} B(0, n)}. \quad (3)$$

Under this system, the FDIC would charge the banks the constant premium, pay them out in the swap, and receive the correct cash flows per period to maintain the fund's solvency probability at

the desired level.

2.2 Incomplete Markets with Annual Contracting

Although arguably arbitrage free, the market for bank failure losses is neither liquid nor complete. In such an incomplete and illiquid setting, there is both non-uniqueness of the risk neutral measure and a lack of bidirectional pricing. Indeed, the ask prices of claims bought will differ from the sale or bid price for the same claim. To determine the periodic floating ask premia κ_n in this setting, we adopt the method of *pricing to acceptability*, i.e. determining the price so that the claim's coherent risk level is acceptable (or formally nonpositive).

2.2.1 Expected Shortfall

To illustrate, suppose we take as the coherent risk measure the expected shortfall at some loss quantile level. Consider a random cash flow Y and a λ quantile level q^λ , i.e. the probability of Y being less than q^λ is λ . Then, the expected shortfall at this level is

$$-E[Y|Y \leq q^\lambda].$$

Note that this expectation is under the statistical probability measure.

In such a setting, if we pay out the nonnegative loss coverage L in period n and charge the premium amount k , then the random cash flow at end of period n is:

$$Y = k(1 + r) - L.$$

Note that for ease of notation, we temporarily drop the subscript n on the cash flows. We show in the appendix that the smallest premia attaining a nonpositive expected shortfall risk level is

given by

$$k = \frac{E [L|L \geq F_L^{-1}(1-\lambda)]}{1+r} \quad (4)$$

where F_L is the distribution function of L under the probability measure P .

The premiums k_n established under expression (4) are annual premia for years $n = 1, \dots, T$. For identical annual distributions, the premia each year would be k and the premia over T years would be Tk . This structure requires contract renewal each year. Alternatively, one can consider a contract where the nonpositive expected shortfall condition at level λ is enforced only over a longer T -year horizon, rather than annually. Under certain conditions, it can be shown that the aggregate premium associated with a T -year horizon contract will be below Tk . In other words, it is better to enter into a long term contract as opposed to a sequence of short term contracts. The charge depends on the contractual period.

There are two additional observations about this risk determined deposit premium. First, it is an expectation under an explicit change of measure. Second, the deposit premia always exceeds the statistical expected loss. To demonstrate the first point, we note that

$$\begin{aligned} k(1+r) &= \frac{1}{\lambda} \int_{F_L^{-1}(1-\lambda)}^{\infty} Lf(L)dL \\ &= \frac{\int_{F_L^{-1}(1-\lambda)}^{\infty} Lf(L)dL}{\int_{F_L^{-1}(1-\lambda)}^{\infty} f(L)dL} \\ &= E \left[L \frac{\mathbf{1}_{L > F_L^{-1}(1-\lambda)}}{\lambda} \right] \\ &= \widehat{E}[L] \end{aligned}$$

where the measure change is

$$\frac{d\widehat{Q}}{dP} = \frac{1}{\lambda} \mathbf{1}_{L > F_L^{-1}(1-\lambda)}.$$

Hence, we see that

$$k = \frac{\widehat{E}[L]}{1+r}.$$

As indicated, this measure-change weighs all high-loss states equally and completely ignores all other states.

Hence, we expect that k exceeds the statistical expectation. This is the second point. To prove this, consider the function

$$H(u) = \frac{\int_u^\infty Lf(L)dL}{\int_u^\infty f(L)dL}$$

where we recall that

$$k = H(F_L^{-1}(1 - \lambda)).$$

Note that $H(0) = E[L] = \mu$, is the statistical expected loss. We next show that H is an increasing function, thereby proving that k will exceed the statistical expectation. Indeed, the derivative of H with respect to u is

$$\begin{aligned} H'(u) &= -\frac{uf(u)}{\int_u^\infty f(L)dL} + \frac{\int_u^\infty Lf(L)dL}{\int_u^\infty f(L)dL} \frac{f(u)}{\int_u^\infty f(L)dL} \\ &= (H(u) - u) \frac{f(u)}{\int_u^\infty f(L)dL} > 0. \end{aligned}$$

The last inequality follows because

$$\begin{aligned} H(u) \int_u^\infty f(L)dL &= \int_u^\infty Lf(L)dL \\ &> u \int_u^\infty f(L)dL \end{aligned}$$

implies $H(u) > u$.

It is also instructive to note that one may, in relatively liquid markets, compute from observed market prices the target level of acceptability attained for any risk measure. Such an exercise is conducted in Cherny and Madan (2008) with respect to writing unhedged options on the *SPX* and the *FTSE* using alternative and improved measures of acceptability over expected shortfall.

2.2.2 Value at Risk (VAR)

Although expected shortfall defines a law invariant convex cone of acceptable risks, it has some drawbacks in practical implementations. First, it has a lack of robustness as noted in Kou and Heyde (2004). Second, the measure is very sensitive to the tail of the loss distribution, and third, it is difficult to estimate. A simpler and more robust measure is provided by value at risk (VAR). Such a measure is less sensitive to the tail of the distribution and is thereby more robust. For this reason, our subsequent simulations will be based on a value at risk measure, but enforced over a long horizon for reasons to be subsequently discussed.

To show that the deposit risk premia under VAR is an expectation under a change of measure, consider again the single period case with loss L for a premium k . The cash flow is

$$Y = k - L.$$

The probability q that $Y > a$ is given by

$$q = F_L(k - a).$$

Setting k to achieve a target level for q , given for example by $a = 0$, determines k as

$$k = F_L^{-1}(q).$$

We show in the appendix that such a deposit premia is an expectation under the change of measure given by

$$\frac{dQ}{dP} = \frac{\mathbf{1}_{L \geq b}}{LE \left[\frac{\mathbf{1}_{L \geq b}}{L} \right]}$$

where one has to solve for b in terms of q .

For reasons similar to those given before with respect to expected shortfall, the deposit premium

k will exceed μ , the statistical expectation of L . Here, large losses $L > b$ have a greater weighting than small losses $L < b$, but the weighting declines as L increases.

As already noted, one may infer parameters of the measure change from market prices by calibrating observed market prices to the expectation under the change of measure. In this way one may transfer knowledge of risk based pricing in existing markets to new markets that are seeking to introduce it.

2.3 Incomplete Markets with Contracting over Long Horizons

The foregoing analysis illustrates the issues that must be considered in designing risk based deposit premium systems with annual contracting. Unfortunately, annual contracting has some difficulties when combined with risk based deposit premiums. These difficulties arise because annual contracting risk based deposit premia exceed the statistical expectation of losses. Hence, on average, the FDIC insurance fund will grow across time. Consequently, rebates must be considered. Rebates complicate the distribution of the final cash flows. In addition, we may wish to embed countercyclical components within the deposit premium system. Countercyclical premium systems are discussed in the next section.

These difficulties with annual contracting lead us to consider dynamic models for the insurance fund over long horizons. The FDIC can be thought of as a governmental "trustee" for the banking industry, whose duty is to manage/oversee a fund for insuring bank failures. The banking industry itself is responsible for funding the entire cost of the insurance and the expenses of the trustee. The trustee's duties are to set fair premiums, to collect the premiums, and to pay the losses due to bank failures. Because losses may exceed premiums in any period, the trustee is also required to collect an initial "maintenance account" (provided by the banking industry) at the start of the Insurance fund which will earn a fair return and from which excess losses will be paid. If the premiums exceed losses in any year, then the trustee should also rebate these excess premiums to the banking industry. Hence one may view the maintenance account as an advance premium

collection in present value terms and this will generally have a tradeoff with the level of subsequent annual premium flows.

The Insurance fund size is set so that the fund's principal and the premiums collected should, with a certain confidence level over a prespecified period of time, not be depleted. Depletion implies fund insolvency. If depleted, then the "trustee" will have to go back to the banking industry to make up any shortfall and to re-establish the principal within the Insurance fund. Because insolvency of the fund would be a costly event, both politically and financially, the confidence level and the time horizon should be set so that insolvency is a rare event. For the sake of discussion (and subsequent simulations), let us suppose that the confidence level is set at 0.95 over a 10– year horizon. That is, the Insurance fund size is set so that the probability of its being depleted over any 10– year horizon is 5 percent.

The complexity of this formulation necessitates the use of simulation, instead of analytical solutions, for the determination of the deposit premium. For analytical convenience, we characterize the net cash flow to the insurance fund, aggregating losses against total premiums, without describing the dynamic evolution of either. Before describing the simulation, however, we first discuss a countercyclical deposit premium system.

3 A Countercyclical Premium System

To simplify the analysis, we assume that the economy is in steady state, where the deposit growth rate equals the spot rate of interest in the economy. Then, given that the premiums, losses and fund size are all linearly related to deposit growth, their growth rates are also equal to the spot rate. This implies that in computing present values, the discount and growth rates cancel. This is equivalent to setting the interest rate equal to zero in the economy discussed in the previous section. We study the countercyclical insurance premium computations in this steady state economy. It is important to emphasize that this steady state assumption is not essential to our analysis or the

conclusions obtained, and it could be relaxed with increased computational effort.

Specifically, in this study, we target a 95% probability of surviving 10 years. Hence, we seek to minimize premiums that are countercyclical and yet meet such a target. Such probability target allows taxpayers to bear some of the default risk of the insurance fund. As indicated by Blinder and Wescott (2001), "reducing the taxpayers' potential exposure all the way to zero is not the appropriate goal of policy."

For the analysis of the risks involved we adopt a discrete time model with annual periods denoted by $n = 0, \dots, N$. We also work in present value terms or equivalently consider a zero interest rate economy. The flat aggregate premium is κ in billions of dollars per year. Denote by C_n the size of the fund, in billions of dollars, at the start of period n , and let L_n be the level of losses, in billions of dollars, paid out by the Insurance fund in period n . We introduce a premium rebate for fund sizes above a benchmark level C . The elasticity of premium rebate with respect to C_n exceeding C is β . We also introduce a rebate with elasticity γ with respect to the aggregate loss level L_n . The annual premium assessed at the end of the year n , P_n , is then

$$P_n = \kappa \left(\max \left(\frac{C_n}{C}, 1 \right) \right)^{-\beta} (1 + L_n)^{-\gamma} \quad (5)$$

For $C_n < C$ and $L_n = 0$ the premium set by equation (5) is the flat rate of κ billion dollars. In other words, κ is the zero-rebate premium in a world when rebates are allowed. It should be noted that this κ is higher than the κ that prevails in a no-rebate system because in states $C_n = C$ and $L_n = 0$, κ must be high enough to allow for rebates in bad states of the world.

We suppose the system starts out with the benchmark level for the fund size of C , and we present the trade-offs in the flat rate $\hat{\kappa}$, the rebate elasticities β, γ that are consistent with long term fund survival measured by a 95% target probability of surviving 10 years.

The only inflows into the fund are the premiums and the only outflows are losses. The fund

size at the start of the next period is then given by

$$C_{n+1} = C_n + P_n - L_n \tag{6}$$

and the insurance fund is declared bankrupt when the fund size C_n reaches a minimal reserve level.

The generation of annual aggregate losses L_n and the operation of the system over time are as follows. We analyze the 10-year survival probabilities by simulating the annual losses for ten years. A random number M_n of failures each year generate the aggregate loss amount. Each of these failures has an associated asset size, A_k for failure by bank k , and loss rate l_k with the k^{th} loss amount being $A_k l_k$ and $L_n = \sum_{k=1}^{M_n} A_k l_k$. The asset sizes are drawn from a stable aggregate distribution of asset sizes, and likewise loss rates are drawn from a stable and independent distribution of loss rates. The loss arrivals are generated by a Poisson process with a constant arrival rate. The constant arrival rate implies that the losses are independent across time. As such, this assumption probably underestimates the likelihood of losses in a downturn, and overestimates the likelihood of losses in prosperous times. On average, however, consistent with our steady state economy assumption, a constant arrival rate provides a reasonable first approximation to a (more complex and realistic) doubly stochastic Poisson process conditioned on the state of the economy.

4 Asset size and loss-rate distributions

4.1 Distributions

The asset sizes that are relevant are those of all banks that are subject to failure in any given year. The loss rates are those that have been experienced in past bank failures. For the US banking system, the distribution of asset sizes has a characteristic property of a substantial number of banks that have assets that are an order of magnitude above the model or most likely asset size.

Such a fat tailed distribution can be well modeled by the parametric class of Frechet distributions. This distribution has two parameters, a scale parameter c_F and a shape parameter a_F with the cumulative distribution function $F(A; c_F, a_F)$ given by

$$F(A; c_F, a_F) = \exp\left(-\left(\frac{A}{c_F}\right)^{-a_F}\right). \quad (7)$$

The associated density is $f(A; c_F, a_F)$ and

$$f(A; c_F, a_F) = \exp\left(-\left(\frac{A}{c_F}\right)^{-a_F}\right) \frac{a_F c_F^{a_F}}{A^{1+a_F}} \quad (8)$$

and the tail of the distribution falls at rate $1 + a_F$ with the consequence that moments exist only for orders less than a_F . Hence the mean, μ_F , is finite for $a_F > 1$ and the variance, σ_F^2 is finite for $a_F > 2$ in which case

$$\mu_F = c_F \Gamma\left(1 - \frac{1}{a_F}\right) \quad (9)$$

$$\sigma_F^2 = c_F^2 \left(\Gamma\left(1 - \frac{2}{a_F}\right) - \Gamma\left(1 - \frac{1}{a_F}\right)^2 \right) \quad (10)$$

The Frechet distribution has a mode A_m below c_F at the point

$$A_m = c_F \left(1 + \frac{1}{a_F}\right)^{-\frac{1}{a_F}} \quad (11)$$

that reflects a positive most likely asset size and yet it has a long tail with substantial probability at large sizes as the density decays at a power law. In this regard it is particularly suited for describing the distribution of asset sizes in a population of many small banks with a few very large ones.

For the loss rate distribution we consider the parametric class of the Weibull distribution. Loss rates are essentially bounded variables for which all moments are finite. This is true for the

Weibull family. This family also has two parameters, a scale parameter c_W , and a shape parameter a_W with the cumulative distribution function $G(L; c_W, a_W)$ given by

$$G(L; c_W, a_W) = 1 - \exp\left(-\left(\frac{L}{c_W}\right)^{a_W}\right). \quad (12)$$

The associated density is $g(L; c_W, a_W)$ and

$$g(L; c_W, a_W) = \exp\left(-\left(\frac{L}{c_W}\right)^{a_W}\right) \frac{a_W L^{a_W-1}}{c_W^{a_W}} \quad (13)$$

The mean, μ_W and variance, σ_W^2 are given by

$$\mu_W = c_W \Gamma\left(1 + \frac{1}{a_W}\right) \quad (14)$$

$$\sigma_W^2 = c_W^2 \left(\Gamma\left(1 + \frac{2}{a_W}\right) - \Gamma\left(1 + \frac{1}{a_W}\right)^2 \right) \quad (15)$$

For $a_W < 1$ this density has a mode at zero representing a most likely loss rate of zero. However, for the case $a_W > 1$ we have a modal loss level L_m below c_W of

$$L_m = c_W \left(1 - \frac{1}{a_W}\right)^{\frac{1}{a_W}}. \quad (16)$$

Furthermore, in this case the probability in the upper tail decreases at a rate that is faster than exponential which makes loss rates near unity relatively uncommon. The shape parameter of the Weibull distribution, a_W parametrizes the behavior of the hazard rate for losses. The hazard rate is the relative probability of a large loss rate to an even larger loss rate. When hazard rates are increasing it gets more and more difficult to get to higher and higher loss rate levels. For the Weibull, with $a_W < 1$ we have decreasing hazard rates while for $a_W > 1$ we have increasing hazard rates.

For the distribution of loss rates we anticipate a positive mode, with an increasing hazard rate

as all attempts are being made to limit losses. Hence the Weibull model with shape parameter above unity is appropriate. We also note that the Weibull model would be inappropriate for asset sizes as it has a fat tail only when its mode is zero while asset sizes have a fat tail with a positive mode. Similarly, the Frechet model is inappropriate for loss rates as it would generate a large number of loss rates above unity. Our empirical tests on the data confirm these conjectures.

Other distributional candidates may also be considered from the prior literature (Madan and Unal, 2004; Unal, Madan, and Guntay (2004), Kuritzkes, Schuermann, and Weiner, 2004). We also provide additional empirical tests of these alternatives with respect to our proposed choices and confirm the adequacy of the model we adopt.

4.2 Empirical Tests of the Distributional Models

For the distribution of asset sizes we use the asset sizes of 8694 banks in the US from the Call Report Data for the year 2000. The loss rate data come from the failed bank data base maintained at the FDIC for the period 1984 – 2000. The number of failures were 1505 of which 32 had a zero loss rate. Our analysis uses the 1473 failed banks with positive loss rates. Table 1 summarizes the time series loss experience of the FDIC for the 1984 – 2000 period. It shows the yearly estimated losses as a percentage of the total assets of the failed banks together with the number of failures for six size categories. Three trends are observable. First, number of bank failures decline as bank asset size increases. For example, only eight banks over \$5 billion asset size failed during the sample period. Second, as asset size increases loss rates decline. For example, while average loss rate for the smallest asset size group is about 25%, for the largest size group this average percentage declines to about 8%. Finally, after 1992, there is a notable decline in the number of bank failures.

The summary statistics on asset sizes and loss rates on failed banks are reported in Table 2. We observe that for the asset size the mean is substantially above the median and in fact also above the upper quartile, suggestive of a highly skewed and fat tailed distribution. This property

is also reflected in the large standard deviation. Furthermore the last percentile relative is 51 times the upper quartile. Hence, the Frechet distribution appears to be an appropriate choice.

We observe that the average loss rate for the 1984 – 2000 period is 21.1% of the assets. The mean and median loss rates are fairly close with the mean in the interquartile range. In addition, the last percentile is well below the unit loss rate and this observation suggests a substantially thinner upper tail. Thus, the Weibull model with a shape parameter above unity, appears a reasonable choice. The differences between the loss rate and asset size data sets are quite marked and provides the early indication that it is not likely that the two data sets come from the same distributional model.

We estimate both the Frechet (equation 8) and Weibull (equation 13) models by maximum likelihood on the asset size data scaled to \$10 billions, for the 8649 banks in the year 2000. The parameter estimates for the Frechet are $a_F = 0.94002$, $c_F = 0.005154$ and the estimates for the Weibull are $a_W = 0.5426$, $c_W = 0.0204$. For graphical convenience Figure (??) plots the histogram of the binned data in steps of \$10 million up to \$500 million. This segment contains 90% of the data.

We observe that the Frechet model fits the data better. It picks up the mode and the long tail quite accurately. The Weibull on the other hand tries to get the long tail and as a consequence is forced to place the mode at zero. The quality of the improvement of the Frechet over the Weibull model is confirmed by Chi Square tests performed in the range of cells with more than 10 observations that go up to asset sizes of \$700 million. The Frechet model could be improved upon in the smaller asset sizes below \$250 million. For the range from \$250 million to \$700 million we have 43 degrees of freedom with the Frechet chi square statistic of 56.69 while the corresponding Weibull value is 416.67. The respective $p - values$ are .0787 for the Frechet and zero for the Weibull.

For the loss rate data, in addition to the Frechet and Weibull models, we employ three other distributions that have been used to describe loss distributions in the literature (Madan and Unal,

2004; Unal, Madan, and Guntay (2004), Kuritzkes, Schuermann, and Weiner, 2004). These are the Gaussian with parameters μ_G, σ_G , the Beta distribution with two parameters α, β and the logitnormal with parameters μ_L, σ_L . The results are presented in Table 3.

Table 3 shows that the Beta and Weibull models dominate the Gaussian, Frechet and Logit Normal as candidates for this distribution. The Weibull reflects a mode and an increasing hazard rate with a fit that is marginally better than the Beta distribution. Figure (??) presents a graph of the histogram of loss rates and the fitted distributions.

We observe from figure (??) the relative closeness of the Weibull and Beta model to each other and the data (displayed as circles). The Gaussian model comes next followed by the Logit normal and the Frechet. This visual ranking of the models is formally confirmed in the χ^2 statistics and corresponding p - *values*. For the latter we use 50 bins with more than 5 observations with the resulting degrees of freedom being 48. The test statistics reported delete the bottom 10% of loss rates and hence we have 38 degrees of freedom.

5 Simulation Results

5.1 Design

A typical simulation run traces the annual progression of the aggregate premium, loss levels and the fund size at beginning of each year for 10 years for 1000 potential paths. The run produces three 10 by 1000 matrices for the aggregate premium, annual loss level and beginning of year fund size. Equation (5) defines the annual premiums. The aggregate annual loss amount is generated by simulating a Poisson number of failures with mean arrival rate of $\lambda = 20$, for each of which we simulate an asset size from the estimated Frechet distribution, and a loss rate from the estimated Weibull distribution, with the aggregate annual loss being sum over the number of losses of the product of the asset sizes and loss rates. The initial fund size for the next year is defined by equation (??) using the premiums and losses that were generated for the year. On any path for

which the funds size reaches the bankruptcy level at the start of some year, the simulation on this path is stopped with the fund size frozen at the bankruptcy level. For the default probability we count the proportion of bankrupt states in the 1000 paths.

For the three components of the loss simulation, the number of failures, the associated asset size and loss rate the details are as follows. For the Poisson number of losses we use the Poisson random number generator from Matlab and generate N_{nm} the number of failures in year n on path m with a constant arrival rate of $\lambda = 20$. This assumption of mean failure rate draws on the failure experience of the FDIC during the post FDICIA period.

For each failure $i \leq N_{nm}$, the asset size we generate a uniform random number $u_{nm}^{(i)}$ for year n on path m and simulate the asset size $A_{nm}^{(i)}$ in accordance with the inverse cumulative distribution method,

$$A_{nm}^{(i)} = c_F \left(-\ln \left(u_{nm}^{(i)} \right) \right)^{-\frac{1}{\alpha_F}}. \quad (17)$$

where, $\alpha_F = 0.94$, $c_F = 0.0051$. To prevent the asset size reaching unreasonable levels we introduce asset cap, p , and modify equation (17) as follows

$$A_{nm}^{(i)} = c_F \left(\left(\frac{c}{p} \right)^{\alpha_F} - \ln \left(u_{nm}^{(i)} \right) \right)^{-\frac{1}{\alpha_F}}. \quad (18)$$

We set p equal to \$500 billion, which is roughly the largest asset size insured bank in the United States in the base year of 2000.

Similarly, for the loss rate associated with failure i , $l_{nm}^{(i)}$ for year n on path m we generate another independent sequence of uniform random variates $v_{nm}^{(i)}$ with the loss rates now given by the inverse Weibull cumulative distribution function

$$l_{nm}^{(i)} = c_W \left(-\ln \left(1 - v_{nm}^{(i)} \right) \right)^{\frac{1}{\alpha_W}}. \quad (19)$$

where, $\alpha_W = 1.7031$, $c_W = 0.2404$.

The aggregate annual loss amount for year n on path m , L_{nm} is then

$$L_{nm} = \sum_{i=1}^{N_{nm}} A_{nm}^{(i)} l_{nm}^{(i)}. \quad (20)$$

The fund size at the start of year n on path m is C_{nm} . The loss amount for the year is L_{nm} .

The premium for the year on this path is

$$P_{nm} = \hat{\kappa} \left(\max \left(\frac{C_{nm}}{C}, 1 \right) \right)^{-\beta} (1 + L_{nm})^{-\gamma} \quad (21)$$

where the policy parameters $\hat{\kappa}, \beta, \gamma, C$ are prespecified. The fund size at the start of the next year is then

$$C_{n+1,m} = C_{nm} + P_{nm} - L_{nm}. \quad (22)$$

The simulation on a path is stopped the first time $C_{n+1,m}$ is below the bankruptcy threshold of half a billion dollars. We assume that the interest earned on the fund balance equals to the expenses of running the insurance fund.

5.2 The Current State

Table 4 reports results for a number of base case alternatives where we assume no rebates are given and a flat premium structure to exist. Case 1 shows that we assume the starting fund size to be 31 billion dollars with total domestic deposits of 3.3 trillion dollars. In addition, we assume premium income to be zero. These initial assumptions are roughly consistent with the state of the insurance fund at FDIC during early 2000s. For a flat premium structure with no countercyclical features β and γ are zero in equation (5). For this setting of the simulation inputs, given the simulated losses, and assuming no premiums are paid during the ten-year period, we find that the default probability in 10 years is 19%.

For a target default probability in 10 years of 5% with no countercyclical features, one may

adjust upward either the fund size or the aggregate premium level. Case 2 shows that keeping the premium level at zero, it takes about doubling of the fund size to \$62.5 billion to reduce the 10 year default probability below 5%. Alternatively Case 3 demonstrates that keeping the fund size at 31 billion dollars one may raise the level of premiums to \$5 billion per year (or an effective assessment rate or .15%) to reduce this 10 year default probability below 5%. Finally, Case 4 shows an intermediate possibility where the fund reserve is raised to 40 billion and the effective assessment rate is increased to .078% to attain the target 5% default probability.

The above simulations employ the same aggregate loss distribution over the ten years as the random number seed is fixed. This distribution is made up of three components cumulated over ten years, and these are the Poisson arrivals, the Frechet assets sizes capped at \$500 billion, and Weibull loss rates each year.

5.3 Countercyclical Trade-offs

We next explore the trade-offs inherent in the design of countercyclical premium systems in Table 5. We consider a number of sample premium schedules around the base scenario of Case 4 in ($C_0 = \$40$ billion fund size and a flat premium level $\kappa = \$2.6$ billion or an effective assessment rate of (EAR) $\$2.6 \text{ billion}/\$3.3 \text{ trillion} = 0.078\%$ that gives a 10–year 5% default probability).

Panel A introduces the base case when no rebates are considered. Panel B introduces rebates based on the level of aggregate losses only. In other words, in terms of Equation (5), we assume $\beta = 0$. In this case, to organize loss rebates the insuring agency needs to decide on the parameter value of γ . Suppose it calls it to be 3.802. This value determines the discount that will be applied to the zero-rebate premium:

$$P_n = \widehat{\kappa}(1 + L_n)^{-3.802}. \tag{23}$$

Every year, depending on the losses on bank failures (L_n), the insuring agency reduces the κ and assesses the premium level. In a given year if there are no losses the banks pay $\widehat{\kappa}$. If losses are

high their premium level is lower.

The parameter value 3.802 implies the following in terms of zero rebate premium. The insuring agency allows a 50% reduction in the annual \$2.6 billion premium level when aggregate losses are \$2 billion. The value of γ that satisfies the equation ($50\% = (1+L)^{-\gamma}$), where $L = .2$, accomplishes this premium reduction.³ Thus $\gamma = 3.802$.

Table 6 Panel B, Column 9 shows that such rebate structure increases the default probability of the fund from 5% to 7.3%. Note that banks will pay into the system the 0.078% EAR annually if there are no losses over the next ten years. However, in this case we allow for losses to be incurred and rebates to be given as a result of costly bank failures. Therefore, the actual EAR varies annually because it is indexed to the aggregate loss levels and on average they should be less than 0.078%. Column 6 shows that on average the EAR is 0.048% with a standard deviation of 0.01%. This reduction in premium payments results in an increase in the default probability of the system.

We now ask what the effect is on the flat rate premium of introducing such a rebate if we wish to bring back the default probability back to 5%. Table 5 shows that the flat rate premium associated with a zero level of losses ($\hat{\kappa}$) with loss-rebate structure in place, now rises to \$6 billion or an effective assessment rate of 0.182%, to maintain the target 10 year default probability at 5%.

Alternatively, the policy choice for γ could be set at a higher level than 3.802 to give rebates at a higher rate. Table 6 shows two such choices where γ is set to be 14.207 and 7.273. These values imply that the insuring agency pays rebates when the loss on failures accumulates to \$0.5 billion and \$1 billion, respectively. In either case default probabilities increase from 7.3% to 9.1% and 8.5%.

One consequence of the loss-rebate only system is that the fund size can keep growing over time to ensure a 5% default probability over ten years. We can adjust the premium structure such

³ We express \$5 billion as .5 because our calculations are in terms of 10s of billion dollars.

that the fund level's growth is slowed down but at the same time the target default probability remains unchanged. One strategy is to collect higher premiums in early years leading to lower levels of the fund size in later years. Such design can be accomplished by augmenting the loss rebate with a rebate system associated with the size of the fund. For this purpose the insuring agency needs to decide on the parameter β . Suppose $\beta = 4.122$ is chosen and no loss rebates are allowed. Then the discount applied to zero capital (and zero loss) premium is

$$P_n = \hat{\kappa} \left(\frac{C_n}{C} \right)^{-4.122}. \quad (24)$$

Panel C shows that when such a structure is in effect the default probability increases to 5.7%. An increase in the zero capital rebate level from \$2.6 billion to \$4 billion reduces the default probability back to 5%.

The parameter value 4.122 implies the following in terms of the starting fund size of \$40 billion. If the fund size rises to \$45 billion, a 12.5% increase, there will be a 38.46% rebate in the \$2.6 billion premium (the \$5 billion excess fund size will be returned to the banking sector roughly in five years). Hence the premium associated with an excess fund size of 12.5% is \$1.6 billion. Such a rebate is organized by $\beta = 4.122$ ($.16 = .26 * \left(\frac{45}{40}\right)^{-\beta}$).

The remainder of the table shows more conservative levels of capital rebates. Finally, Panel C brings together both capital and loss rebates and identifies the default probabilities and premium structure.

Tables 4 and 5 underscore an important fact about countercyclical deposit insurance system. For the system to enjoy the luxury of the rebates needed for countercyclical deposit insurance pricing, either the insurance fund must have a super surplus or the premium structure need to be increased drastically. For example, In Table 6 we observe that a loss and fund-size rebate system requires the EAR to increase from 0.078% (no rebate system EAR) to 0.107%. At a level of \$3.3 trillion insured deposits, this EAR implies that the banking system needs to pay an additional

\$0.9 billion on average per year over a ten-year period to enjoy the benefits of a countercyclical premium system. Given that the EAR for the no rebate system is 0.078%, this additional premium represents an average of 35% increase in aggregate premium level.

The limitations of these findings are as follows. First, the loss distribution we use is estimated using the FDIC experience during the 1984-2000 period. We realize that this period may not reflect the loss distribution faced by the FDIC in the next decade. One important consideration is the prompt corrective action (PCA) provision of the FDICIA, which requires regulatory intervention in advance of insolvency. Such mandate can substantially reduce expected costs (Blinder and Wescott, 2001). However, we use this period to allow for the possibility of adverse macro shocks experienced in the 1980s.

Second, we assume the same loss rate distribution for large and small bank failures. However, Bennett (2000) shows that cost of resolving small banks is much higher than the cost of resolving large banks. Hence, loss rates and failed bank asset size are correlated. Our simulations do not allow for such association and apply the same loss rate distribution for small and large bank failures.

Therefore, our results should be taken as the upper limit for a risk-based and countercyclical insurance premium structure that attains a target 95% survival probability of the insurance fund over ten years.

6 Conclusion

This paper provides a mechanism for exploring deposit insurance premium systems that are both responsive to relief in times of crisis and that distribute excess fund reserves, while ensuring viability of the system. We simulate the performance of such a deposit insurance premium system, calibrating the distributions of assets sizes and loss rates. The asset size and loss distributions are modeled well by the Frechet and Weibull families, respectively. The paper shows that the

benefits of a countercyclical rebate system does not come free. The system should be ready to pay a substantial cost to finance a countercyclical rebate system.

7 Appendix:

7.1 Premia under Acceptability Pricing for incomplete and illiquid markets

The expected shortfall at quantile level λ with the probability of an outcome less than q^λ is

$$E [Y|Y \leq q^\lambda(Y)]. \quad (25)$$

Let $k(1+r) = a$. To determine $q^\lambda(Y)$ we first obtain the distribution function of Y .

$$\begin{aligned} P [Y \leq y] &= P [a - L \leq y] \\ P [L \geq a - y] &= 1 - F_L(a - y) \end{aligned}$$

Hence,

$$\lambda = P [Y \leq q^\lambda(Y)] = 1 - F_L(a - q^\lambda(Y))$$

and,

$$q^\lambda(Y) = a - F_L^{-1}(1 - \lambda) \quad (26)$$

Plugging 26 in 25, we obtain

$$E [a - L|a - L \leq a - F_L^{-1}(1 - \lambda)] \quad (27)$$

$$a - E [L|L \leq F_L^{-1}(1 - \lambda)] \quad (28)$$

Noting that $28 \geq 0$ and $a = k(1+r)$, we obtain

$$k_n = \frac{E[L|L \geq F_L^{-1}(1-\lambda)]}{1+r}.$$

7.2 Premia under VAR (Madan and Yor)⁴

We wish to equate pricing attaining a target value at risk with pricing under a change of measure.

We consider the special case of paying out a positive random variable X and charging a price k with the cash flow

$$Y = k - L.$$

We determine k such that the probability that $Y > 0$ is targeted to be q . Hence we must have that

$$F_L(k) = q$$

where F_L is the distribution function of L .

We also write that the price is

$$k = F_L^{-1}(q)$$

and we wish to see this as an expectation under a change of probability or

$$k = E[\Lambda_q L]$$

for a change of probability Λ_q .

It seems natural to look for

$$\Lambda_q = \Psi_q(L)$$

⁴ We thank Mark Yor for his contribution in the development of this appendix.

where we take

$$\Psi_q(x) = \frac{\mathbf{1}_{x>b}}{c_2x}$$

Thus

$$E[\Psi_q(L)L] = \frac{1 - F_L(b)}{E\left[\frac{\mathbf{1}_{L>b}}{L}\right]}$$

Denote $Y = \frac{1}{L}$ then we have

$$\eta(b) = \frac{P(Y < \frac{1}{b})}{E[Y\mathbf{1}_{Y<1/b}]}$$

or equivalently that

$$\frac{1}{\eta(b)} = \frac{E[Y\mathbf{1}_{Y<1/b}]}{P(Y < \frac{1}{b})}$$

Let $\beta = 1/b$ and we seek β such that

$$\frac{E[Y\mathbf{1}_{Y<\beta}]}{P(Y < \beta)} = \frac{1}{F^{-1}(q)}$$

Define

$$G_Y(\beta) = \frac{E[Y\mathbf{1}_{Y<\beta}]}{P(Y < \beta)}$$

Let $f(y)$ be the density of Y . We may then write

$$G_Y(\beta) = \frac{\int_0^\beta yf(y)dy}{\int_0^\beta f(y)dy}$$

that starts at zero and goes to $E[Y]$ as β tends to infinity. The derivative is

$$\begin{aligned} G'_Y(\beta) &= \frac{\beta f(\beta)}{\int_0^\beta f(y)dy} - G_Y(\beta) \frac{f(\beta)}{\int_0^\beta f(y)dy} \\ &= \frac{f(\beta)}{\int_0^\beta f(y)dy} (\beta - G_Y(\beta)) > 0 \end{aligned}$$

and so we increase to $E[Y]$ and can solve all cases for which

$$\frac{1}{F^{-1}(q)} < E[Y]$$

We expect that targets for q are such that

$$F^{-1}(q) > E[X]$$

and hence

$$\frac{1}{F^{-1}(q)} < \frac{1}{E[X]} < E\left[\frac{1}{X}\right] = E[Y]$$

Hence all relevant cases can be solved.

8 References

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Table 1: Total assets, loss as a % of total assets, number of bank failures

| | Over 5 B | 1-5 B | 500M-1B | 100-500M | 50-100M | Under 50M |
|------|------------------|------------------|-----------------|-------------------|------------------|-------------------|
| 1984 | 39,957(7%) 1 | - - | 513(1%) 1 | 1,345(13%) 7 | 419(16%) 7 | 1,197(23%) 64 |
| 1985 | 5,279(7%) 1 | - - | - - | 1,075(9%) 5 | 454(22%) 6 | 1,928(25%) 108 |
| 1986 | - - | 1,589(14%) 1 | 598(23%) 1 | 1820(26%) 10 | 1468(25%) 21 | 2164(27%) 112 |
| 1987 | - - | 1,200(0%) 1 | 501(13%) 1 | 3,284(26%) 15 | 1,251(24%) 18 | 2,993(27%) 168 |
| 1988 | 18,162(11%) 1 | 10,949(13%) 4 | 7,717(9%) 12 | 10,788(12%) 53 | 3,560(15%) 51 | 3,280(26%) 159 |
| 1989 | 7,181(22%) 1 | 6,932(22%) 3 | 4,373(16%) 6 | 8,739(14%) 37 | 1,685(23%) 25 | 2,695(25%) 135 |
| 1990 | - - | 4,144(9%) 2 | 1,950(12%) 3 | 5,703(24%) 26 | 1,488(19%) 22 | 2,455(20%) 116 |
| 1991 | 45,591(3%) 4 | 9,146(16%) 7 | 4,619(22%) 6 | 5,943(23%) 23 | 1,535(18%) 21 | 1,629(21%) 66 |
| 1992 | 7,269(10%) 1 | 23,704(5%) 9 | 3,421(15%) 6 | 8,304(10%) 32 | 1,456(18%) 20 | 1,334(18%) 54 |
| 1993 | - - | - - | 936(13%) 1 | 1,389(20%) 7 | 582(21%) 8 | 621(20%) 25 |
| 1994 | - - | - - | - - | 1,217(12%) 7 | 77(23%) 1 | 111(10%) 5 |
| 1995 | - - | - - | - - | 635(10%) 3 | 77(13%) 1 | 31(29%) 2 |
| 1996 | - - | - - | - - | - - | 114(19%) 2 | 68(25%) 3 |
| 1997 | - - | - - | - - | - - | - - | 26(19%) 1 |
| 1998 | - - | - - | - - | 375(60%) 1 | - - | 53(8%) 2 |
| 1999 | - - | 614(0%) 1 | - - | 115(9%) 1 | 157(27%) 2 | 61(10%) 3 |
| 2000 | - - | - - | - - | 114(11%) 1 | 239(6%) 3 | 38(5%) 2 |

Table 2: Asset Size and Loss Rate Summary Statistics

| | Asset Size (\$Billions) | Loss Rate (%) |
|--------------------|-------------------------|---------------|
| Mean | .751 | 21.10 |
| Standard Deviation | 10.0 | 12.97 |
| Minimum | .0013 | .0053 |
| Maximum | 584 | 93.94 |
| Median | .084 | 19.63 |
| Lower Quartile | .042 | 11.64 |
| Upper Quartile | .188 | 28.80 |
| First Percentile | .008 | .3084 |
| Last Percentile | 9.67 | 56.42 |

Table 3: Results on distributional models for loss rates

| Model | Parameter 1 | Parameter 2 | χ_2 | p-value (df=38) |
|--------------|---------------------|-------------------|----------|-----------------|
| Gaussian | $\mu_G = 0.2066$ | $\sigma = 0.1319$ | 76.06 | 0.00024 |
| Beta | $\alpha = 1.8454$ | $\beta = 6.7546$ | 49.86 | 0.0942 |
| Weibull | $\alpha_W = 1.7031$ | $C_W = 0.2404$ | 48.13 | 0.1256 |
| Frechet | $\alpha_F = 0.9814$ | $C_F = 0.109$ | 570.46 | 0 |
| Logit Normal | $\mu = -1.5182$ | $\Sigma = 0.7175$ | 122.17 | 0 |

Table 4: Base case alternatives with no rebate system

| | Case 1 | Case 2 | Case 3 | Case 4 |
|---------------------------------|--------|--------|--------|--------|
| Fund size (\$billion) | 31 | 62.5 | 31 | 40 |
| Domestic deposits (\$trillion) | 3.3 | 3.3 | 3.3 | 3.3 |
| Effective assessment rate (%) | 0 | 0 | 0.15 | 0.078 |
| Aggregate premium (\$billion) | 0 | 0 | 5 | 2.6 |
| 10-year default probability (%) | 19 | 5 | 5 | 5 |

Table 5: Trade-offs in the design of countercyclical premium system

| | γ | β | LR | CR | κ | Zero Rebate EAR | Avg. Eff. EAR | EAR σ | DP (%) |
|---------|----------|---------|----|------|----------|--------------------|------------------|-----------------|-----------|
| Panel A | 0 | 0 | 0 | 0 | 2.6 | 0.078 | 0.078 | 0 | 5 |
| Panel B | 3.802 | 0 | 2 | 0 | 2.6 | 0.078 | 0.048 | 0.01 | 7.3 |
| | 3.802 | 0 | 2 | 0 | 6 | 0.182 | 0.112 | 0.02 | 5 |
| | 14.207 | 0 | .5 | 0 | 2.6 | 0.078 | 0.021 | 0.01 | 9.1 |
| | 14.207 | 0 | .5 | 0 | 15 | 0.455 | 0.122 | 0.03 | 5 |
| | 7.273 | 0 | 1 | 0 | 2.6 | 0.078 | 0.035 | 0.01 | 8.5 |
| | 7.273 | 0 | 1 | 0 | 9 | 0.273 | 0.123 | 0.03 | 5 |
| Panel C | 0 | 4.122 | 0 | 1 | 2.6 | 0.078 | 0.061 | 0.01 | 5.7 |
| | 0 | 4.122 | 0 | 1 | 4 | 0.121 | 0.078 | 0.02 | 5 |
| | 0 | 1.8132 | 0 | 0.5 | 2.6 | 0.078 | 0.068 | 0.01 | 5.3 |
| | 0 | 1.8132 | 0 | 0.5 | 3.0 | 0.092 | 0.075 | 0.01 | 5 |
| | 0 | 1.2275 | 0 | 0.25 | 2.6 | 0.078 | 0.07 | 0.01 | 5.2 |
| | 0 | 1.2275 | 0 | 0.25 | 2.8 | 0.085 | 0.074 | 0.01 | 5 |
| Panel D | 7.273 | 1.813 | 1 | 0.5 | 2.6 | 0.078 | 0.034 | 0.01 | 8.5 |
| | 7.273 | 1.813 | 1 | 0.5 | 11 | 0.33 | 0.107 | 0.02 | 5 |