

Advances in Asset and Liability Management: Modeling Insurance and Pension Liabilities in an Integrated Enterprise Risk Management Framework¹

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One of the biggest and best protected silos in the risk management business has been the liability side of the insurance business. In many insurance firms, the pricing of new policies by actuaries and the risk management of entire enterprise are viewed as the same thing. Today, the Chinese wall surrounding actuarial risk assessment is being rapidly broken down. Best practice risk management demands an integrated approach to risk assessment at insurance companies and pension funds. “Integrated risk” means the simultaneous assessment of risk on the entire balance sheet, including both assets and (insurance) liabilities, and spanning all of the risk disciplines: credit risk, market risk, liquidity risk, asset and liability management, actuarial (insurance) risks, and capital adequacy or solvency. This post explains how to implement a fully integrated assessment of insurance and other risks.

For decades, the asset side and liability side of insurance companies have shared the same warm relationship as North and South Korea. Non-actuaries on the asset side of the organization were repeatedly rebuffed by actuaries when the non-actuaries expressed concern about risk management that seemed solely focused on the asset side of the organization. For example, it’s common practice for insurance companies to use a risk management system for the asset side of the organization, but the liabilities of the firm are not loaded into the asset-side risk system. This lack of integrated asset and liability management is extraordinary and extremely surprising from the perspective of other sides of the financial services business. The actuaries would typically reply, “You can’t possibly understand valuation and risk assessment of insurance liabilities; only we have the training to do such a task.” Much the same arguments were made by

¹ An earlier version of this note appeared as Chapter 38, “Valuing Insurance Policies and Pension Obligations,” in *Advanced Financial Risk Management* (van Deventer, Imai and Mesler, John Wiley & Sons, 2004).

analysts of collateralized debt obligations, with dire consequences in the 2007-2009 credit crisis.

Emerging best practice in the insurance and pension fund management business correctly points out two very important reasons why an integrated approach, combining the asset and liability sides of the organization, is logical and natural:

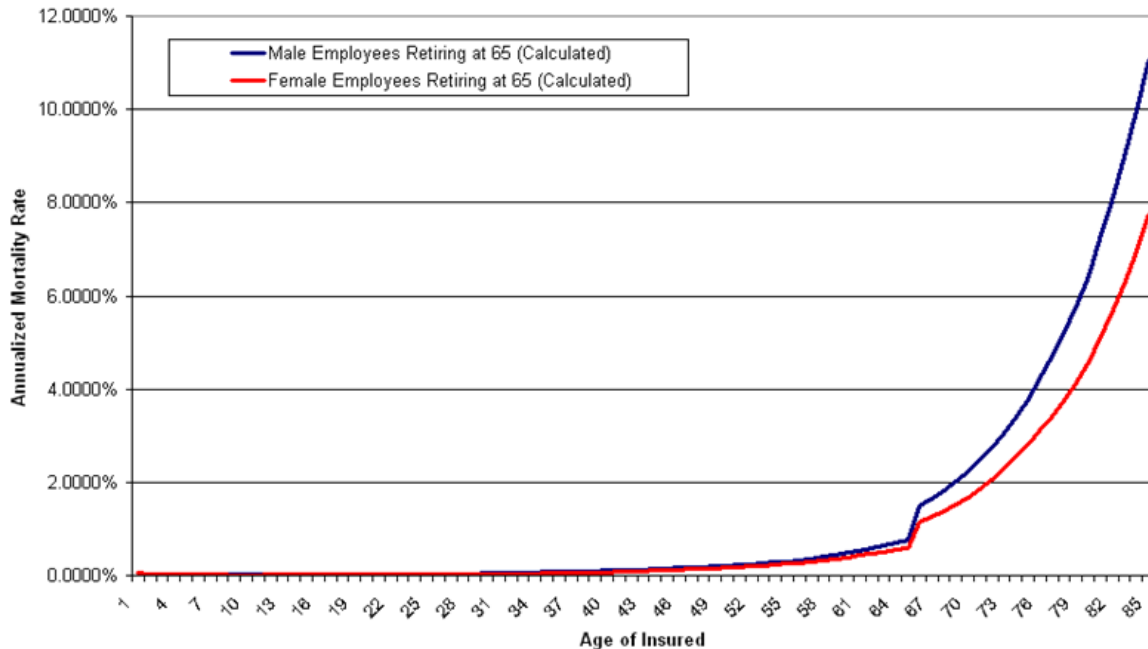
- Macro economic factors can have a very important impact on both sides of the balance sheet, and any risk analysis which ignores these links is needlessly dangerous. Three letters make this obvious: “AIG” or “PMI,” private mortgage insurance.
- Actuarial mathematics is not something obscure or misunderstood in finance. In fact, actuarial mathematics is at the very heart of “reduced form credit risk modeling.” The two disciplines stem from the same work done by respected actuary D.R. Cox.

Life Insurance: Mortality Rates vs. Default Probabilities

The analogy between the mortality rate on a life insurance policy and the default probability on a bond of a particular issuer is strong. The chart below shows annual mortality rates for healthy male and female employees retiring at age 65:²

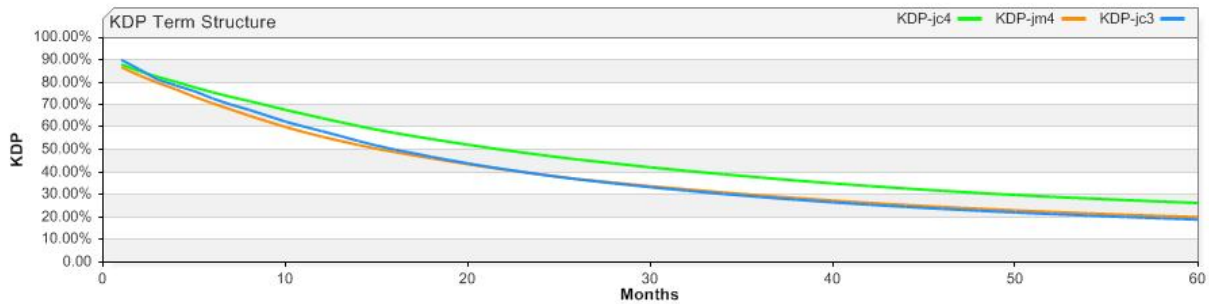
² Society of Actuaries RP-2000 Actuarial Tables.

Mortality Rates for Healthy Employees Retiring at Age 65 from the SOA RP-2000 Actuarial Tables



Like default or bankruptcy, mortality on a life insurance policy is a zero/one kind of event. The fundamental approach to measuring mortality risk was proposed by D.R. Cox in 1972.³ The “Cox process” that he developed is in fact the same “Cox process” used in reduced form modeling in the Jarrow-Turnbull [1995], Jarrow [1999, 2001] and Duffie and Singleton [1999] credit models that were reviewed in Chapter 17 of *Advanced Financial Risk Management* (van Deventer, Imai and Mesler, John Wiley & Sons, 2004). The mathematics is identical. The “term structure” of mortality rates shown above is exactly the same as the term structure of default probabilities that one can observe in “mortality rate” for Enron that modern reduced form credit risk technology would have predicted on November 30, 2001. “KDP” standards for Kamakura default probability. The term structure of default or “corporate mortality” is given for three different models, the third and fourth generation reduced form models and a Jarrow-Merton hybrid model where the Merton default probability is an explanatory variable in the reduced form framework:

³ “Regression Models and Life Tables,” *Journal of the Royal Statistical Society*, B34, 187-220, 1972.



The only superficial difference between these two curves (mortality rate and default rate) is due to differences in human and corporate biology. Human mortality asymptotically approaches 100% as age increases. That is not necessarily true with corporations, even in Enron's case. The analytical differences between human and corporate mortality stem more from differences in what variables drive the predicted mortality rates, rather than differences in how these explanatory variables are determined and how they are used to simulate risk forward.

The other strong parallel between life insurance and credit risk is the progressive use of logistic regression to determine the probability of the event (bankruptcy or mortality) with more precision than one would get from a simple table (a transition matrix with default probability by ratings grade or from an annuity or mortality table). The days are long past when best practice life insurance policy pricing would depend only on basic variables like age, sex and smoker/non-smoker. A mortality model which reflects the individual's current medical condition will obviously outperform one that doesn't. This is true even though the standard annuity or actuarial table contains highly summarized data like the following:

Table 4-5

Male RP-2000 Rates				
Age	Employees	Healthy Annuitant	Combined Healthy	Disabled Retiree
41	0.001142		0.001142	0.022571
42	0.001215		0.001215	0.022571
43	0.001299		0.001299	0.022571
44	0.001397		0.001397	0.022571
45	0.001508		0.001508	0.022571
46	0.001616		0.001616	0.023847
47	0.001734		0.001734	0.025124
48	0.001860		0.001860	0.026404
49	0.001995		0.001995	0.027687
50	0.002138	0.005347	0.002138	0.028975
51	0.002288	0.005528	0.002449	0.030268
52	0.002448	0.005644	0.002667	0.031563
53	0.002621	0.005722	0.002916	0.032859
54	0.002812	0.005797	0.003196	0.034152
55	0.003029	0.005905	0.003624	0.035442
56	0.003306	0.006124	0.004200	0.036732
57	0.003628	0.006444	0.004693	0.038026
58	0.003997	0.006895	0.005273	0.039334
59	0.004414	0.007485	0.005945	0.040668
60	0.004878	0.008196	0.006747	0.042042
61	0.005382	0.009001	0.007676	0.043474
62	0.005918	0.009915	0.008757	0.044981
63	0.006472	0.010951	0.010012	0.046584
64	0.007028	0.012117	0.011280	0.048307
65	0.007573	0.013419	0.012737	0.050174
66	0.008099	0.014868	0.014409	0.052213
67	0.008598	0.016460	0.016075	0.054450
68	0.009069	0.018200	0.017871	0.056909
69	0.009510	0.020105	0.019802	0.059613
70	0.009922	0.022206	0.022206	0.062583
71		0.024570	0.024570	0.065841
72		0.027281	0.027281	0.069405
73		0.030387	0.030387	0.073292
74		0.033900	0.033900	0.077512
75		0.037834	0.037834	0.082067

Indeed, when one of us recently took out a life insurance policy, the following information was requested in the application:

- May insurance at another company be discontinued or changed if this policy is issued?
- Is there an application pending at another insurance company?
- Do you plan to travel outside the US, Canada, Europe, Hong Kong or Australia/New Zealand for business or pleasure?
- In the past two years have you participated in aeronautics, powered racing or competitive vehicles, skin or scuba diving, mountain climbing, rodeos or competitive skiing?
- Have you been convicted of or pleaded guilty to (in the last 5 years) moving violations, drunk driving, reckless driving?
- Do you fly as a pilot or co-pilot?
- Have you ever been convicted of a crime?
- Are you a member of the armed forces? Do you intend to become a member?

- Social security number
- Do you smoke?
- Who is your doctor?
- When did you last see him? Why?
- What is the age of your parents?
- Have you had an EKG in the last 5 years? Why?
- Any operations?
- Shortness of breath?
- Declined for life insurance?
- Occupation?
- Any injuries?
- Any diseases of
 - Nervous system
 - Blood
 - Digestive tract
 - Kidneys
 - Joints or muscles?
- Diabetes?
- Cancer?
- Physical defects?
- AIDS?
- Drugs?
- Cancer in family?
- Has weight changed by more than 15 pounds?

These questions were posed because they are potentially important explanatory variables in predicting mortality. In addition to the client-supplied information above, a nurse hired by the insurance company made a personal visit to collect the following additional medical information from the potential insured:

- Height
- Weight
- Chest Measurement
- Waist Measurement
- Urine Test
- Blood Test
- Electro Cardiogram

A recent paper by Guizhou Hu⁴ shows how mortality rate prediction can be improved by incorporating explanatory variables like these that are much more detailed than those in

⁴ "Mortality Assessment Technology: A New Tool for Life Insurance Underwriting," undated memorandum, BioSignia, Inc., Durham, North Carolina.

mortality tables (age, male or female, smoker or non-smoker). Among the variables he mentions are the following:

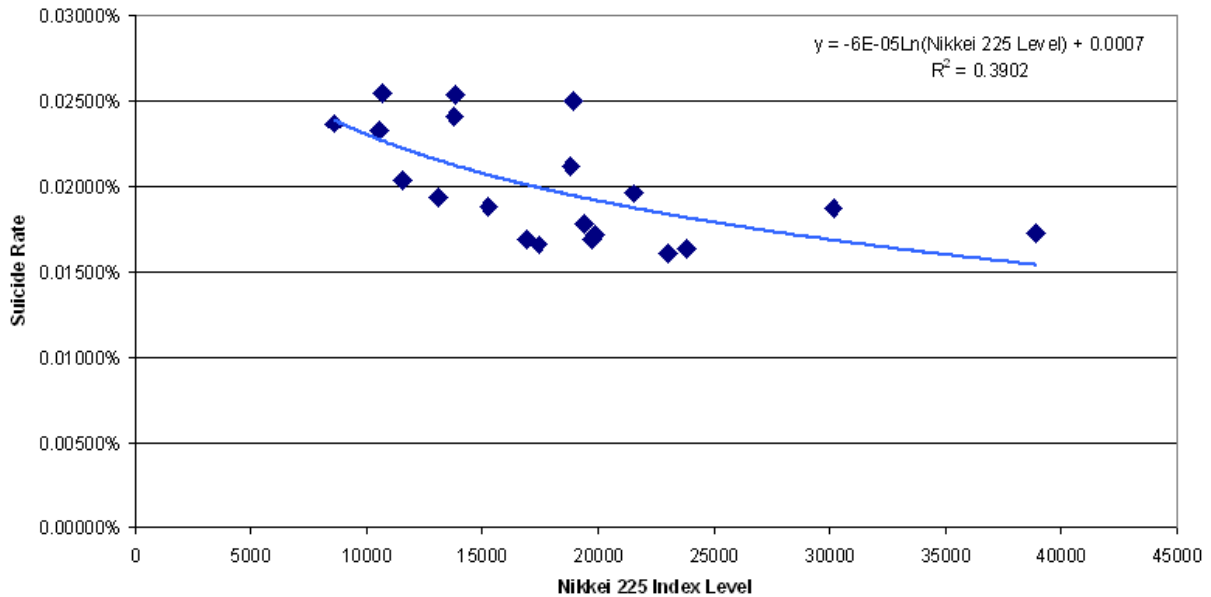
- Systolic blood pressure
- Total cholesterol
- Body mass index
- Smoker or non-smoker
- Albumin
- Physical activity
- Light
- Moderate
- Heavy
- Income

All of these input variables are specific to the insured, just like the accounting variables and stock price inputs used in reduced form credit models like those offered by Kamakura Risk Information Services. We revisit the issue of macro-economic variables and their impact on all insureds or all corporate borrowers later in this piece.

Cyclicalities in Default Probabilities and Mortality Rates

Logistic regression can also confirm the importance of cyclicalities in mortality just as it does (as required by the New “Basel II” Capital Accords) for default. Any Japanese banker can tell you that the mortality rate of his co-workers is much different when the Nikkei stock index is below 10,000 yen than it was when it was nearly 39,000 yen. The graph below illustrates that fact clearly using data from Japan both before and after the collapse of the Japanese bubble starting at the end of 1989:

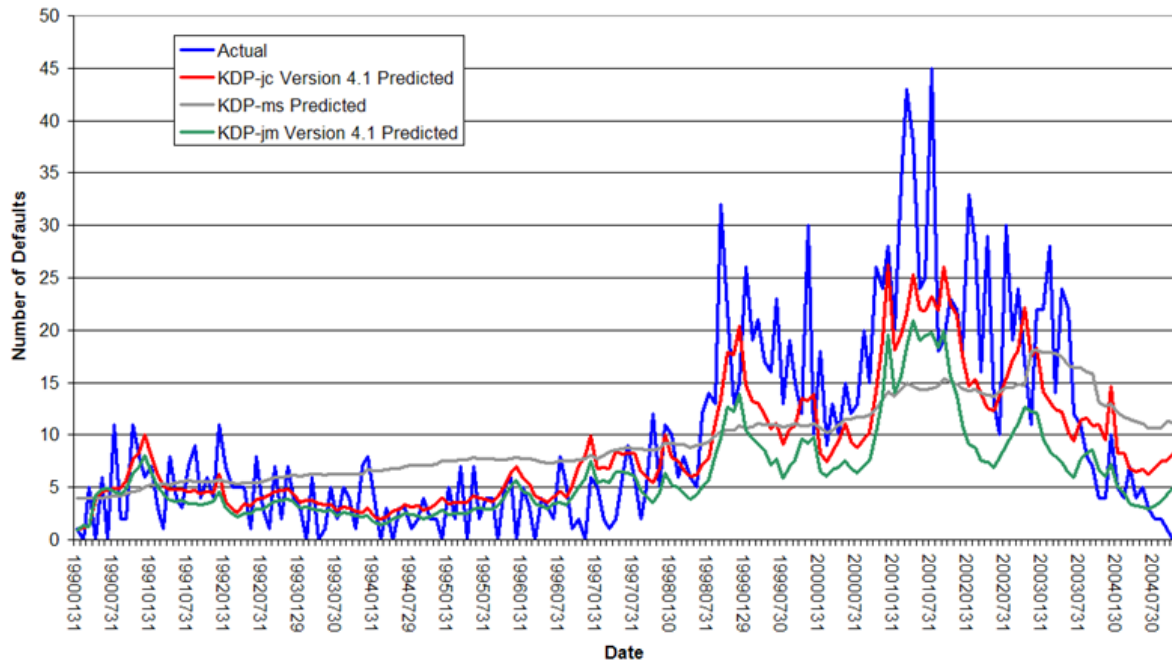
**Suicide Rate in Japan as
Function of Nikkei 225 Closing Level,
1984-2003**



It is well known that Russian mortality rates deteriorated rapidly immediately after the collapse of the former Soviet economy, only to improve in later years as the new capitalist economy took root and grew. Mortality rates are also very powerfully affected by long term trends triggered both by disease (like AIDS) and advances in medical science.

Similarly, there is strong cyclicality in corporate defaults as this graph shows. It compares (in sample) the actual and predicted number of public firm defaults in North America from 1990 to October 2004 according to Kamakura Risk Information Services:

Actual Versus Predicted Defaults, North America, 1990-2004 for KRIS Version 4.1



These analogies between default and mortality have been well known to actuaries for decades, as this quote⁵ indicates:

“While everything in this chapter will be stated as insurances (sic) on human lives, the ideas would be the same for other objects such as equipment, machines, loans and business ventures. In fact the general model is useful in any situation where the size and time of a financial impact can be expressed solely in terms of the time of the random event”

What has changed in recently years is that this conceptual understanding has been translated into action by the insurance industry practitioners of best practice risk management.

Valuing Life Insurance Policies

⁵ *Actuarial Mathematics*, by Newton L. Bowers, Hans U. Gerber, James C. Hickman, Donald A. Jones, and Cecil J. Nesbitt, published by the Society of Actuaries, 1986. Chapter 4, page 85.

Basic life insurance policies are very closely related to digital credit default swaps that pay a fixed dollar pay-off upon the occurrence of a “corporate mortality event.” The protection “buyer” on the digital default swap called a life insurance policy is (usually) the insured. The insured is also the “reference name” on the digital life insurance policy. The counterparty on the digital life insurance policy is the life insurance company. The amount paid on the digital life insurance policy is the policy amount. The only significant difference is the maturity—term life insurance policies, which have an explicit maturity, are almost literally identical to digital default swaps. A “whole life” policy runs until mortality or until cancelled. Ignoring the cancellation provisions, it’s like a infinite maturity digital default swap except that we know the term structure of mortality/default probabilities approaches 100% in time.

A very talented actuary might respond to a digital CDS analogy like this: “True but this is not how actuaries look at this. We apply probabilities of death in our modeling so that a little bit of each person dies each year.” I once made this argument myself to my good friend Dr. David Shimko. “Can’t we just lay out the mortality rates for all insureds, argue that diversification applies, and just work with expected or average cash flows on the life policies?” David replied, “I can prove you wrong in one word: AIDS.” A Japanese risk expert would have offered different proof: “Go back to the table of mortality rates as a function of the Nikkei. It’s obvious that the mortality rates are not constant.”

What about cancellation privileges? This is a real option of the insured that can be exercised rationally or irrationally like the prepayment on a mortgage. It is rational to cancel when (a) the mortality risk of the insured becomes lower than that implied in the current policy, allowing the insured to reinsure at a lower rate, or (b) when the insured becomes so wealthy that he no longer needs insurance (he saves money by not paying the insurance company costs embedded in the policy and effectively “self insures.” “Irrational” cancellation (like irrational prepayment) is usually for a good reason unknown to the insurance company. The financial condition of the insured, for example, may have deteriorated so much that he can’t afford the premiums, even though he needs the insurance.

What about “investment options” on a universal or variable life policy? Many actuaries argue that these investment options are so complex that no non-actuary could possibly understand them. We know from finance theory that an option to switch from one instrument at its market price to another instrument at its market price has zero value, since there is no benefit to the options holder (if we ignore transactions costs). Many of the investment options in a life insurance policy have this nature. To the extent there is a real benefit to the holder of the insurance policy from the investment options offered, we can value them using standard “no arbitrage” techniques for valuing derivatives and other securities.

One of the key benefits of life insurance to the buyer is the ability to defer taxes on the investment income of a whole life insurance policy (the “inside build-up of cash value”). This tax deferral is of course a real benefit to the buyer of the insurance policy relative to making the identical investments in the name of the insured. The value of this benefit to the buyer of the insurance policy depends on the state of the tax code now and in the future, the source and variability of income now and in the future, and the default probability of the insurance company. Clearly, valuing these tax benefits with precision is a challenge, and one that we’re happy to defer to another time.

Pension Obligations

Pension obligations have a very interesting mix of terms that combine part of the life insurance problem with more traditional financial instruments. One of the “assets” of a pension fund in an economic sense is this the highly likely on-going stream of current payments for future pension benefits from both employees and their employers. The present value of this cash inflow from any one employee, Mr. Jones, depends on three factors: the mortality probability of Mr. Jones (he could be unlucky enough to pass away before leaving employment and receiving pension benefits), the probability that Mr. Jones will change jobs and end up contributing to a different pension fund, and the probability of default of Mr. Jones’ employer. The liabilities of a pension fund, the future payments to the beneficiaries of the fund, depend on the mortality rate of the pension beneficiary, just like a life insurance policy. The difference is that the payments are made in the opposite circumstance of a life insurance policy—they are paid while the beneficiary is still living. In this sense, the future stream of pension payments are each a “digital default” swap of sorts again, except the key probability used in valuation is the probability of no mortality prior to the payment, not the probability of bankruptcy/mortality relevant to valuing a life insurance policy.

For defined benefit pension plans, which are our focus in this section, there are normally a complex set of rules or circumstances which determine when and how pension benefits are increased over time. Inflation is a key macro factor driving this liability of the pension fund, at the same time that inflation can have a major impact on the value of the assets of the pension fund. This is exhibit number 1 as to why integrated asset and liability management is essential.

Similarly, even with government pension insurance, there is the probability that the pension fund defaults on its obligations to the pension beneficiaries. Both of these complexities can be dealt with using a combination of the life insurance analysis above and standard techniques for the valuation of complex securities. In particular, the increased use of the logistic regression technique and reduced form modeling approach allows for comprehensive credit-adjusted mortality-adjusted risk management of both the assets and liabilities of a pension fund.

Property and Casualty Insurance

Property and casualty insurance differs from life insurance in two dimensions: the term of the insurance contract is typically shorter, and the events being insured are quite diverse, ranging from auto insurance to catastrophic weather-related events. Aside from events that are acts of nature, like weather and earthquakes, logistic regression again plays a powerful role in both pricing and risk management of liabilities on a property and casualty insurance contract. Obviously the price of an auto insurance policy depends on factors like

- The age of the driver
- The sex of the driver (in some jurisdictions)
- The health of the driver (if the data were available)
- The income of the driver (again, if the data were available)
- The past driving history of the driver
- The amount of driving to be done (and the possibility that the true amount is not disclosed)
- The type of car being insured (i.e. a bus or a sports car)
- The value of the car being insured
- The place in which the car will be stored (a deserted urban parking lot in a high crime area or a locked garage in a gated community)
- The place in which the car will be driven (i.e. downtown New York City or rural France)

In addition, it is an empirical question whether macro-economic factors have an impact on both the probability of an insured event and the payout on the occurrence of the event. Clearly, we know from the statistics on new auto sales that new sales of cars decline in bad times so the average age of all automobiles being driven increases during recessions. This in turn would be expected to affect the probability of a claim and the payout on the claim. For this reason, depending on the nature of the insured event, it will often be the case that macro factors drive the probability of an insurance payout just like they affect the probability of default in reduced form credit models. When this is the case, it is essential that these macro factors be incorporated in policy pricing and total balance sheet risk management. If one does not do this, the risk of providing the policy can be dramatically underestimated. That certainly turned out to be the case with private mortgage insurance, where the highly correlated risk of these home price-related insurance policies was compounded by investing the premiums in mortgage-backed securities. If macro factors are relevant to liability cash flows, they need to be explicitly incorporated in analysis of both the asset side and liability side cash flows to properly measure the correlation in the occurrence of insured events and the “fat tails” of the loss distribution.

Best Practice Risk Management

In very many insurance companies and pension funds, there is a sharp division between the investment or asset side of the organization and the “insurance” or liability side of the organization, although this is less true than it was in the past. Finance experts dominate the investment function and the liability side is dominated by insurance experts and actuaries. This contrasts sharply with risk management at banks and securities firms where “asset and liability management” on a joint basis has been standard for 30 years. Thanks to insights made obvious by the 2007-2009 credit crisis and the collapse of the Japanese bubble, risk analysis is now focused on a broader array of risk factors like home prices and commercial real estate prices, not just interest rates and foreign exchange rates.

Looking at the risk on only one side of the balance at a time is truly dangerous because common macro factors drive the risk on both sides of the balance sheet. If you are looking at only half the picture, it is impossible for management, the board of directors and regulators to have an accurate view of total risk.

Fortunately, as we have seen in this note, the mathematics of credit risk models is taken directly from insurance expert Cox. There is literally no difference in the mathematical approach, so there are only emotional and political reasons for the long-standing divide between the actuaries and the investment side of the organization when it comes to risk management. The best practice insurance companies are solving this problem by moving actuaries to the investment side of the organization and investment managers to the insurance side of the organization. Integrated risk management teams that have both actuaries and finance experts are being established and a common view of integrated risk is widely recognized as necessary and desirable. We expect that the historical walls down the middle of insurance companies and pension funds will break down rapidly in the years ahead.

What risk management questions need to be answered by top management at insurance firms and pension funds? They are the same questions that were posed in our blog “A 4 Question Pass/Fail Test on Risk Management for CEOs and Members of the Board of Directors,” www.kamakuraco.com, on April 27, 2009 (redistributed on www.riskcenter.com, April 28, 2009). We challenged insurance companies and other financial institutions to answer these questions:

Question 1: What happens to the market capitalization and net income of the firm if any of these risk factors change: home prices, foreign exchange rates, commercial real estate prices, stock index levels, interest rates, commodity prices?

Question 2: Using an insider's knowledge of the assets and liabilities of the firm, both "on balance sheet" and "off balance sheet," what is the best estimate, monthly for the next ten years, of the probability that the firm will fail in each of these 120 monthly periods?

Question 3: Using only information available to an outsider, what is the best estimate of the probability of the failure of the firm in both the short run and the long run?

Question 4: If the firm is able to answer Questions 1, 2, and 3, what hedging position is necessary to insure that the macro factor sensitivity of the firm and default probability of the firm reach the target levels set by the Board of Directors?

How should macro-factor driven asset and liability cash flows be simulated forward to measure risk on an integrated basis? In exactly the same way as we explained in this blog post: "Credit Portfolio Models: The Reduced Form Approach," Kamakura blog, www.kamakuraco.com, on June 5, 2009 (redistributed on www.riskcenter.com on June 9, 2009). This multi-period credit-adjusted and mortality-adjusted simulation produces more than just the answers to the four questions above. It's a framework for completely integrated balance sheet and financial-accrual based net income simulation, along with the generation of simulated cash flows, liquidity risk assessment and measures of capital adequacy. As in banking, solvency regulations and capital adequacy rules in insurance can be starkly different from the way a talented risk manager would evaluate safety and soundness. Nonetheless, all of the inputs to regulatory capital requirements can be simulated in a consistent framework, even when the mortality rates for statutory capital requirements are different from the best estimates of mortality that actuaries can produce. This has exact parallels in Basel II's focus on using ratings based estimates of one year default rates for measuring bank capital adequacy, even though the credit crisis of 2007-2009 has made it clear that a reliance on ratings is less accurate than more modern approaches.

Best practice risk managers in the insurance industry and pension fund industry are moving full speed ahead in this direction.

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